

An Introduction to the Application of Interpretive Structural Modeling*

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Interpretive Structural Modeling is an emerging methodology which appears to be very useful as an aid to individuals and small groups in developing an understanding of complex situations. This paper presents an introduction to the fundamental concepts and operations of the methodology and reports on the results of two exercises conducted with a group of graduate students who had minimal mathematical training. The first exercise involved the structuring of personal values and was intended to acquaint the individuals with the methodology. The second was a group exercise focusing on barriers to investment in the central city, a subject of substantive interest to the participants. The results of the exercises demonstrate the utility of the methodology for capturing and communicating individual and group perceptions regarding complex issues.

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I. INTRODUCTION

The term Interpretive Structural Modeling (ISM) is used here to refer to the systematic application of some elementary notions of graph theory in such a way that theoretical, conceptual, and computational leverage is exploited to efficiently construct a directed graph, or network representation, of the complex pattern of a contextual relationship among a set of elements.

The mathematical foundations of the methodology can be found in various reference works (e.g., *Structural Models: An Introduction to the Theory of Directed Graphs* by Harary and Cartwright¹). The philosophical basis for the development of the ISM approach has been presented in Reference 2, and the conceptual and analytical details of the ISM process are outlined in Reference 3. The methodology has been implemented in a man/machine interactive environment in such a way that human users are responsible for making subjective judgments while the computer is employed in an unobtrusive manner for book-keeping and for performing and displaying the results of simple logical operations. Preliminary versions of the computer pro-

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grams were available in the Fall of 1973, when the first computer-aided experiments in the use of the methodology were begun.

This paper reports on two exercises which were conducted in a classroom setting with a group of eight graduate students in the Department of City and Regional Planning at The Ohio State University. The undergraduate degrees of the students involved were scattered among economics, geography, political science, and natural resources. The first exercise involved the structuring of personal values and was intended to acquaint the individuals with the methodology. The second was a group exercise focusing on barriers to investment in the central city, a subject of substantive interest to the participants.

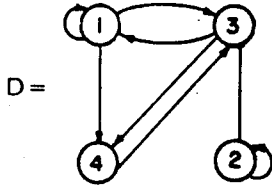
In the following, an introduction to the basic concepts and operations involved in the ISM technique is followed by a description of each of the exercises, including the presentation and discussion of the results.

II. DIGRAPHS, BINARY MATRICES, AND INTERPRETIVE STRUCTURAL MODELING

The process of Interpretive Structural Modeling is based upon the one-to-one correspondence between a binary matrix and a graphical representation of a directed network. The fundamental concepts of the process are an "element set" and a "contextual relation." The element set is identified within some situational context, and the contextual relation is selected as a possible statement of relationship among the elements in a manner that is contextually significant for the purposes of the enquiry. (Examples of element sets and relations are given later in this

section.) The elements correspond to the nodes on a network model, and the presence of the relation between any two elements is denoted by a directed line (or link) connecting those two elements (nodes). In the equivalent binary matrix representation, the elements are the contents of the index set for the rows and columns of the matrix, and the presence of the relation directed from Element i to Element j is indicated by placing a 1 in the corresponding intersection of Row i and Column j .

Some basic terminology and mathematical operations will be introduced now by illustrating the process of developing a hierarchical restructuring of a given directed graph, or "digraph." Consider a system composed of four elements, $S = \{1, 2, 3, 4\}$, and one relation, $R = \rightarrow$, for which the following digraph has been constructed:



A binary matrix representation of this digraph is

$$A = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

This matrix is termed the *adjacency matrix* of D , and is constructed by setting $a_{ij} = 1$ wherever there is an arc in D directed from Element s_i to Element s_j , and by setting $a_{ij} = 0$ elsewhere.

Elements s_j is said to be *reachable* from Element s_i if a path can be traced on D from s_i to s_j . By convention, an element s_i is said to be reachable from itself by a path of length 0. The *reachability matrix*, M , of a digraph is defined as a binary matrix in which the entries m_{ij} are 1 if Element s_j is reachable from Element s_i ; otherwise $m_{ij} = 0$. It can be shown that the reachability matrix can be obtained operationally from the adjacency matrix by adding the identity matrix and then raising the resulting matrix to successive powers until no new entries are obtained. That is:

$$M = (A + I)^n$$

where n is determined such that

$$(A + I)^{n+n} < (A + I)^{n+1} < (A + I)^{n+1}$$

Here all mathematical operations are Boolean; hence each successive powering operation preserves the entries of the previous power, and matrix equality or inequality can be determined on the basis of an entry by entry comparison. For the above example $n = 2$ and the reachability matrix is:

$$M = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{matrix}$$

There is a correspondence between the notion of reachability in graph theory and the notion of transitivity in binary relations. If the relation R used to establish the connectivity in the digraph D is transitive—that is, aRb and bRc imply that aRc —then the

matrix M represents the *transitive closure*, D_t , of the digraph D . (This interpretation of the reachability matrix is an essential aspect of the ISM process, which will be elaborated shortly.)

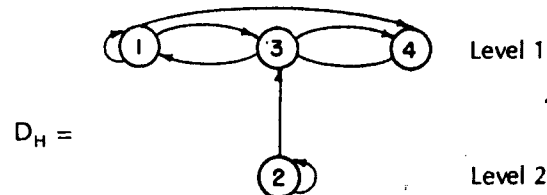
The utility of the reachability matrix is that it can be used to develop a hierarchical restructuring of the digraph. To do this, a *reachability set*, R_i , is defined for each element, $s_i \in S$, as all of those elements which are *reachable* from s_i , that is, all of those elements whose columns have an entry of 1 in row i . This definition is stated mathematically as: $R_i = \{s_j \mid m_{ij} = 1\}$, where m_{ij} is the entry in the i th row and j th column of M . Further an *antecedent set*, A_i , is defined for each element, s_i , as all of those elements which can *reach* s_i , that is, all of those elements whose rows have an entry of 1 in column j . Stated mathematically: $A_i = \{s_j \mid m_{ji} = 1\}$. The *intersection* of A_i and R_i , denoted by $A_i \cap R_i$, consists of all of those elements which are common to both A_i and R_i . Those elements, s_i , for which $A_i = A_i \cap R_i$ are *not* reachable from any of the remaining elements of S , and hence can be denoted as *bottom level elements*.

Continuing with the example, the A_i , R_i , and their intersections are given by:

i	R_i	A_i	$A_i \cap R_i$
1	1, 3, 4	1, 2, 3, 4	1, 3, 4
2	1, 2, 3, 4	2	2
3	1, 3, 4	1, 2, 3, 4	1, 3, 4
4	1, 3, 4	1, 2, 3, 4	1, 3, 4

Here it can be seen that the single Element 2 is in the bottom level. By striking from the matrix M the row and column corresponding to Element 2 and repeating the above process, it is possible to determine the bottom level elements of the reduced system, which are actually the *second* level elements of the original system. This operation is equivalent to simply removing the Element 2 wherever it appears in the above table; thus by inspection, it can be seen that the second level of the example system consists of all the remaining elements: 1, 3, and 4.

This process has rearranged and partitioned the original element set $S = \{1, 2, 3, 4\}$ into *hierarchical components*: $S_H = \{2; 1, 3, 4\}$. (Note that the hierarchical reordering process could have proceeded from the *top down* by identifying the elements for which $R_i = A_i \cap R_i$.) The hierarchically reordered digraph, D_H , then becomes:



What has been gained from this process is a partitioning and rearrangement of the original digraph in such a way that additional information about the system—that is, the presence and content of levels—is transmitted through the format of the display. Although the value of this additional information may appear to be negligible in the case of the above example, situations will appear later in this paper in which hierarchical rearrangement greatly enhances the communicability of the digraph.

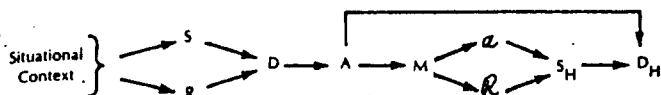


FIGURE 14-1. A Symbolic Digraph of the Process of Extracting a Digraph From a Situational Context and Then Producing a Hierarchical Display. Symbols are Defined in the Text. Arrows Denote the Presence of the Relation: "...is used to determine..."

A symbolic digraph summarizing the above process is shown in Figure 14-1. It should be evident from a review of the previous discussion that it is possible to bypass the construction of the initial digraph, D , and construct its equivalent adjacency matrix directly, continuing then through the process to obtain D_H . Whether to carry out such an exercise beginning with the construction of D or A depends upon the preferences and degree of mathematical preparation of the modeler. In either case, for a large element set and/or a complex pattern of relationships, extensive and laborious operations could be involved. Interpretive Structural Modeling (ISM) was conceived as a procedure for avoiding as much as possible the necessity for knowing—and the burden of performing—the related calculations. A further advantage of removing this computational burden is that the conceptual utility of structural modeling can then be made available to mathematically unsophisticated users. The ISM methodology employs the digital computer for bookkeeping and performing logical operations in the process of directly synthesizing a digraph from a specified set of elements and a relation. Some additional assumptions and related limitations are required, however, and these will now be elaborated.

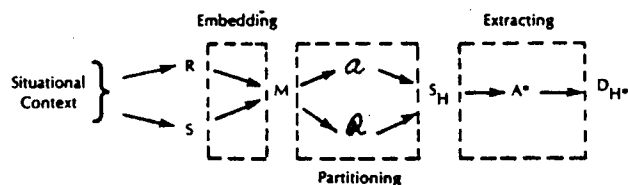


FIGURE 14-2. A Symbolic Digraph of the Principal Operations Involved in the Interpretive Structural Modeling Process. (Symbols and operations are elaborated in text.)

The procedure outlined above and summarized in Figure 14-1 requires the initial specification of a digraph or its equivalent binary matrix. The noteworthy contribution of the ISM procedure, illustrated in Figure 14-2, is that it operates without such an a priori knowledge of the structure. The process is initiated by specifying an element set and a transitive relational statement. An "embedding" operation is then performed, in which advantage is taken of the ability to make transitive inferences within the reachability matrix to guide the systematic interrogation of the user with regard to the presence or absence of the relation between pairs of elements. The user is requested to respond to a query of the form: "(Is (element s_i) (related to) (element s_j)?)", where the verbal constructs appearing in the parentheses are those appropriate to the particular context, e.g., "Does activity 23 precede activity 14?", "Does the number of electric cars on the road affect the number of power plants in an urban area?" A digital computer is employed to keep track of the responses supplied by the user (which are explicit entries to the reachability

matrix), to provide implicit transitive inferences based upon previous responses, and to generate an efficient ordering of subsequent queries.

Following the embedding operation, the resulting reachability matrix is used to partition the element set according to the procedure outlined above. After rearranging and partitioning the reachability matrix to correspond to the hierarchical reordering of the element set, it is then possible (as explained in detail in Reference 3) to systematically examine "diagonal expansions" of M in order to discover the least number of connections which are required to form an adjacency matrix, A^* , whose transitive closure would be equal to the constructed reachability matrix. This operation is referred to as "extracting". (Both partitioning and extracting operations are performed in the computer, with no intervention required by the user.) The corresponding minimum-edge digraph, D_H^* , provides an efficient, hierarchically ordered display of both the direct responses and the indirect transitive inferences resulting from the embedding operation. The superscript asterisks are added to A and D_H in Figure 14-2 to denote that they have been constructed using the ISM process, and specifically that all redundant links have been removed from the digraph. Depending upon the context of the exercise, however, redundant links may be essential to convey the full meaning and pattern of the relation. For this reason, the ISM process provides for a "comparison" operation in which the user examines the result of the mathematical operations and heuristics of the process and introduces modifications or corrections to the digraph. And finally, a "substitution" operation consists of the introduction of elaborative text, interpretive symbols, or additional graphical embellishments which will make the final "interpretive structural model" comprehensible to a wider audience.

ISM is intended for use when it is desired to utilize systematic and logical thinking to approach a complex issue and then to communicate the results of that thinking to others. The objective is to expedite the process of creating a digraph, which can be converted to a structural model, and then inspected and revised to capture the user's best perceptions of the situation. The entire process has been implemented for use in a man/machine interactive environment in such a manner that the user can concentrate on substantive concerns in order to make subjective

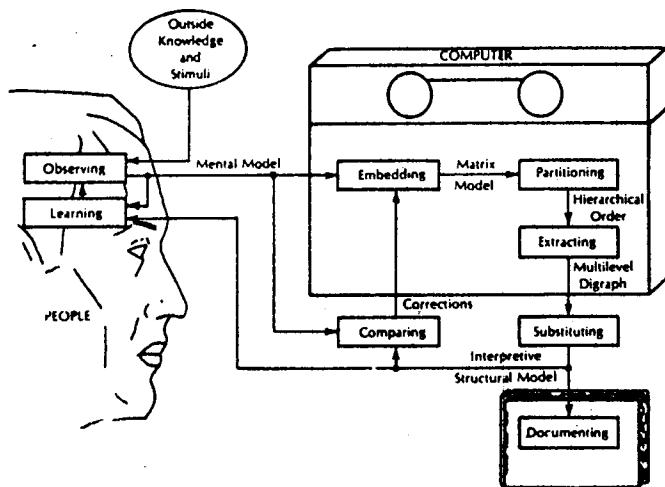


FIGURE 14-3. Functional Representation of Interpretive Structural Modeling, Showing Potential for Man/Machine Symbiosis

judgements regarding the presence or absence of the relation between pairs of elements, and the computer is assigned the task of bookkeeping and routine calculations. The entire ISM process can be represented graphically as shown in Figure 14-3. This figure is an interpretive structural model of the process of building interpretive structural models; it has been generated from Figure 14-2 by substituting simple verbal statements for the mathematical symbols and by introducing the additional operations of "substituting" and "comparing." Note that Figure 14-3 can be used to describe the process to individuals who have had little or no mathematical training.

III. INDIVIDUAL EXERCISE: ORDERING PERSONAL VALUES

The first exercise was designed to introduce the students to the concepts and operations underlying the process. The conceptual context was chosen to have a high degree of personal relevance to the individual students and to involve nontrivial

issues, while not being so extensive as to preclude hand calculations. The element set chosen was Lasswell's eight value categories⁴, and the relation was taken as "... is more important to me than ...," as listed in Figure 14-4. (This may or may not be a transitive relation, a point which enhances the conceptual utility of this exercise, as will be discussed below.) The

Elements (The Eight Lasswell Value Categories):

1. Wealth
2. Skill
3. Enlightenment
4. Power
5. Affection
6. Respect
7. Well-being
8. Rectitude

The Relation:

"(Value i) is more important to me than (value j)."

FIGURE 14-4. The Elements and Relation Employed in the Individual Exercise

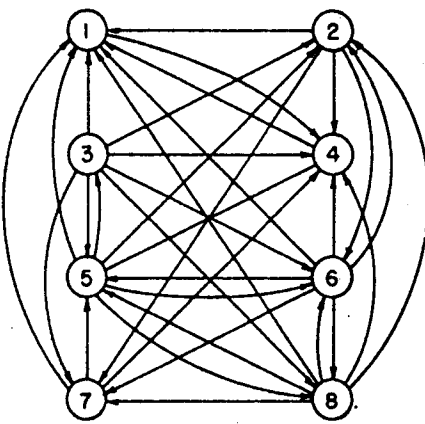
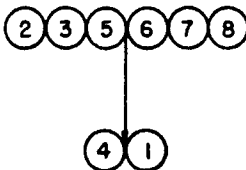
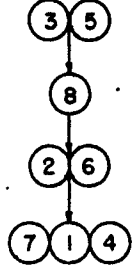
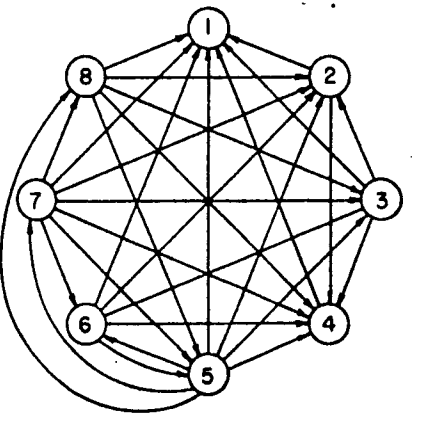
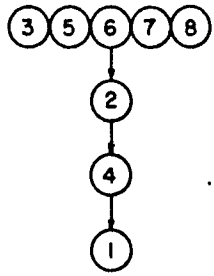

Student	Exercise 1a: A Priori Construction of Digraph and Manual Restructuring, Figure 14-1.		Exercise 1b: Computer Aided Construction of Digraph Using ISM Process, Figure 14-2.
	Initial Digraph	Hierarchically Restructured Digraph	
A			
B			

FIGURE 14-5. Illustrative Examples of Student Responses to Value Structuring Exercise. Element Numbers Correspond to the Values Listed in Figure 14-4. Arrow Denotes the Relation "...is More Important Than..."

... value
... impor-
... or may
... concep-
... v.) The

students were asked to individually create an a priori digraph using these conditions and then to determine the corresponding hierarchical ordering using the operations summarized in Figure 14-1. They were then given access to a computerized version of the ISM procedure and asked to repeat the exercise within an environment characterized by Figure 14-3. This section will present and discuss the results of these activities.

Results for two of the students are presented in Figure 14-5. The initial digraphs shown in Column 1 of the figure clearly show one construction strategy, in which the elements were laid out in a geometric pattern to aid the manual process of systematically making the pairwise comparisons. These digraphs appear very complex, but when they are converted to binary matrices and subjected to a hierarchical reordering calculation, following the procedure outlined in Figure 14-1, considerable simplification results, as shown in Column 2 of Figure 14-5. In these figures two elements neither of which is "more important" than the other, symbolically shown as $\text{①} \leftrightarrow \text{②}$ or equivalently as ①② , are interpreted as being "equally important." (This convention was established prior to the exercise.) The hierarchical digraphs are arranged such that all equally important elements appear on the same level; elements in the higher levels are more important than elements in the lower levels.

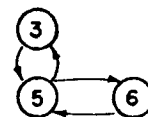
Sequential elimination was another strategy employed by some students for constructing the initial digraph. In this strategy the list of elements was scanned for the most important element or group of elements, which would then be placed in the highest level and eliminated from the set; the reduced set would then be scanned for the next most important elements, and so on. Three graphs which were constructed in this manner are shown in Column 1 of Figure 14-6; Column 2 of the same figure presents the digraphs which were constructed by the same students by using the computerized ISM process. Note that for the students using this construction strategy there was relatively little difference between the initial and the ISM structures, in contrast with the previous cases illustrated in Figure 14-5.

This exercise provided an experiential context within which to discuss several considerations related to the process of interpretive structural modeling. First the comparison between the hand calculations and the computer-aided operations demonstrated the considerable computational leverage provided by the ISM process, both in terms of time required to process the structure—two to four hours for the manual approach, five to ten minutes for the ISM approach—and in reduced requirements for methodological knowledge—the computer performs all the necessary computations.

In addition, the exercise provided "objective" data for use in a general discussion of the concept of "values," and the implications of differences between individual and collective value hierarchies. The general concept of values has been discussed in many recent works, including the previously referenced work by Lasswell⁴, so no further discussion will be presented here. The question of the legitimacy of a hierarchy of values appears to be closely related to the notion of "transitivity," which is central to the ISM process, and so is worth considering more closely.

The relation "... is more important than ..." is logically transitive. It is perhaps an open question, however, whether individual perceptions of this relation are transitive; that is,

although the relation is transitive, individuals may not be! Consider, for example, the following construct, taken from the initial digraph of Student A (Figure 14-5):



This student presumably inserted double arrows wherever he perceived the pair of values to be equally important. Why then did he leave out the arrow (relation) directed from value 6 to value 3? There is clearly a transitively inferred link by virtue of

Individual

d

4-2

Student	Initial Digraph	ISM Digraph
C		
D		
E		

FIGURE 14-6. Further Examples of Student Responses to Value Structuring Exercise. Elements as listed in Figure 14-4. Upper Elements are "More Important" Than Lower Elements.

the path from 6 to 5 to 3. If this student deliberately left out the link from 6 to 3, thus intending that Value 3 (enlightenment) was strictly more important to him than Value 6 (respect), then he was being intransitive. In such a case, the operation of taking a transitive closure will insert a link from 6 to 3, and the resulting hierarchical digraph will not portray the original intent of the student. (Note that there is no intrinsic reason for human beings to be transitive, but it may be of value to know when they are and when they aren't.)

The ISM process makes use of the property of transitive inference to reduce the number of explicit pairwise comparisons which must be made among the elements, and thus never gives the user the opportunity to enter an intransitive response. Although the process forces the user initially to produce a logically consistent, transitive digraph, intransitive substructures may be introduced through the subsequent "correction" process, which allows the user to explicitly insert or delete elements and/or links. Thus, it is important that users of the ISM process understand both the value and the implications of the transitivity assumption.

Having collected a set of individual rankings of a particular element set, as contained in Figures 14-5 and 14-6, it is of some interest to consider the possibility of aggregating these rankings to produce a collective ranking (or hierarchy). One simple way to do that in this case is to assign weights, w_{ij} , for the i th element according to its relative position in individual j 's hierarchy. By summing over the individuals, a collective score, W_i , can be assigned to each element:

$$W_i = \sum_{j=1}^{n_j} w_{ij}$$

This can then be used to establish relative positions (or "ranks"), R_i , in a collective hierarchy. For the seven students who participated in this exercise, the collective hierarchy of values is displayed in Figure 14-7. A measure of individual consistency with this ranking can be obtained by computing an individual consistency score, I_j , as the average difference between the individual's ranking and the collective ranking:

$$I_j = \frac{1}{8} \sum_{i=1}^8 [w_{ij} - R_i]$$

For the seven participating individuals, the worst consistency score was an average difference of 1% in the relative positions along the hierarchy, the best consistency score was 3%, and the average was 1 1/4%. This is a remarkable consistency, and suggests that these students form a fairly homogenous group.

Rank, R_i	Element	Score, W_i	
7.5	5. Affection	44	(Most Important)
7.5	7. Well-Being	44	
6	8. Rectitude	41	
5	6. Respect	39.5	
4	3. Enlightenment	37	
3	2. Skill	19.5	
2	4. Power	14.5	
1	1. Wealth	12.5	(Least Important)

FIGURE 14-7. Combined Rank and Scores for the Lasswell Value Categories Based upon Responses of Seven Graduate Planning Students

Type of Obstacle	Specific Obstacle	Downtown Area Investment		Inner-City Poverty Area Investment		Gray Area Investment	
		Residential	Commercial	Residential	Commercial	Residential	Commercial
Neighborhood Conditions	High crime rates and insecurity	■	●	■	■	■	●
	Vandalism	—	—	■	■	●	●
	Poor quality public schools	—	●	■	●	■	●
	Traffic congestion	■	●	—	●	—	■
	Poor quality of other public services	●	—	●	●	●	●
	Abandoned buildings	—	—	■	●	—	—
	Poor property maintenance	—	—	■	■	■	■
	Concentrated poverty	●	●	■	■	—	■
	Messy neighborhood appearance	—	—	●	●	●	●
	Non-child-oriented environment	●	—	●	—	—	—
Market Conditions	White withdrawal	●	■	■	■	■	■
	Downward income shift—low demand	—	●	■	■	■	■
	Fragmented land ownership	■	—	■	—	■	—
	Excess supply of housing	●	—	●	—	■	—
	Need for subsidized housing	●	—	■	—	—	—
	Superior suburban competition	●	■	—	—	■	■
Cost-Increasing Conditions	High land costs	■	■	●	●	●	●
	High construction costs	■	●	●	●	●	●
	High property taxes	●	●	●	●	●	●
	High insurance costs	—	—	■	■	—	—
	High security costs	●	●	■	■	●	●
	Government delays	■	●	■	●	—	—
	High borrowing costs and nonavailability of financing	—	—	■	■	■	—
	High land assembly costs	●	●	—	—	—	—
	Excessive uncertainty	■	—	■	■	■	■

KEY

■ Crucially important ● Significant, but usually not crucial — Of minor or negligible importance

Prepared by Real Estate Research Corporation*

FIGURE 14-8. Relative Significance of Specific Obstacles in Hindering Private Investment in Different Parts of Central Cities.

Raising the issue of the significance and legitimacy of such a strategy for generating a collective perception led to a spirited discussion among the participants. Only one of the students involved in the exercise seriously questioned the conceptual legitimacy or wisdom of structuring personal values. In fact, the possibility that one value may be instrumental to another leads to some interesting questions regarding the utility of a simple preference ordering. Further, since the ranking and aggregating approach used here ignores relative intensity of preferences, there are some possible theoretical flaws that should be considered. A full discussion of these points is beyond the scope of this paper. More sophisticated measuring and aggregating techniques are available, however, and the interested reader is referred to the psychometric literature, e.g., Reference 5.

IV. GROUP EXERCISE: BARRIERS TO INVESTMENT IN THE CENTRAL CITY

The second exercise was conducted the week after the first and involved the participation of the full group of eight students in using the computerized version of the ISM methodology to collectively create a network representation of the pattern of

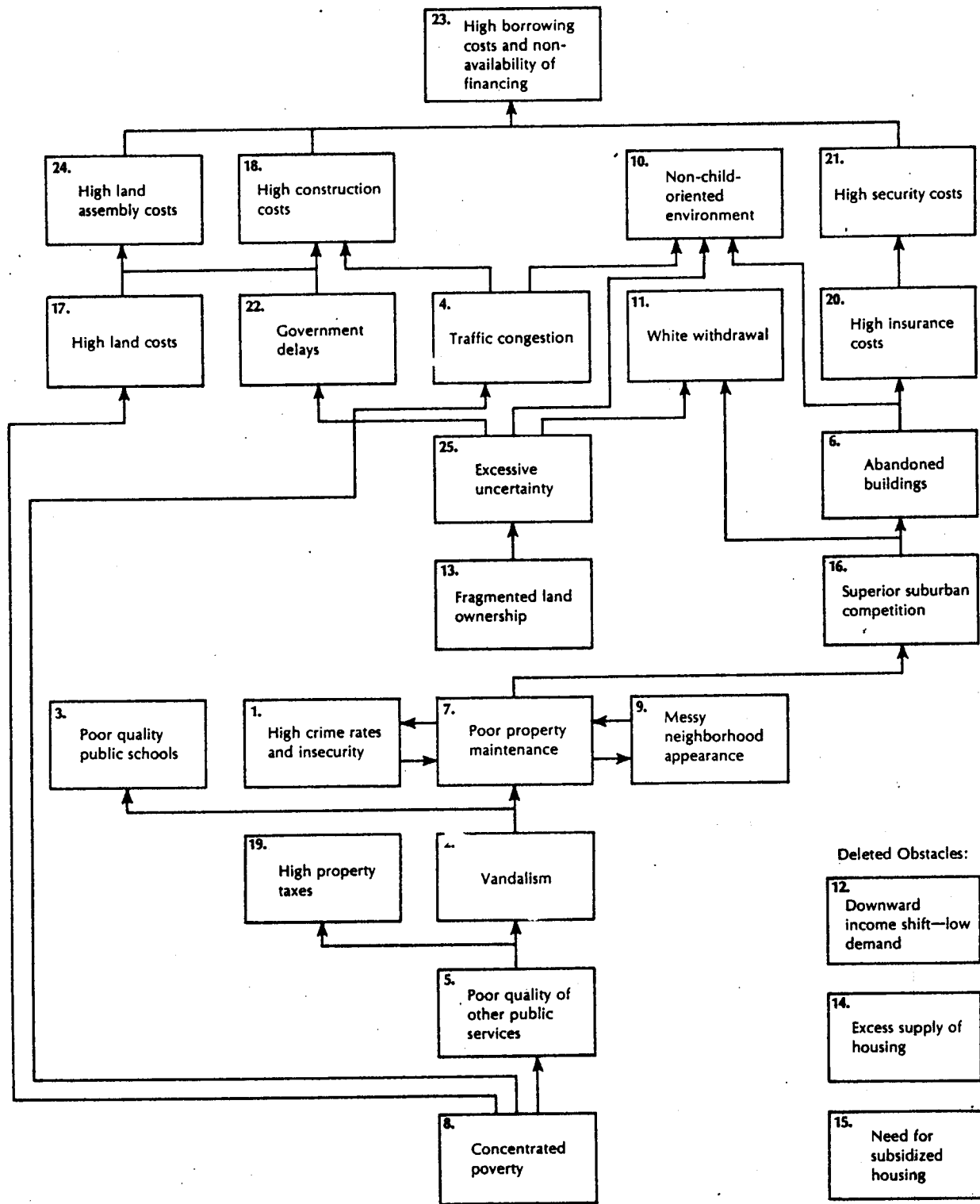


FIGURE 14-9. Interdependence of Obstacles to Investment in the Columbus, Ohio, Central Business District; as Perceived by a Group of Ohio State University Planning Students. (Lower level obstacles intensify or aggravate higher level obstacles. The list of obstacles and their interpretation were taken from Reference 6.)

interaction among a set of obstacles to investment in the central city. The set of obstacles was taken from a comprehensive paper by Anthony Downs⁶ which served as the substantive background for the exercise. The primary objectives of the following discussion are to describe the conditions under which the group exercise was conducted, to present some statistics regarding the operation of the process, and to illustrate the type of "model"

that is produced.

The element set used is described in detail in Reference 6. Figure 14-8 is taken from that paper and summarizes Downs' perceptions of the relative importance of the various obstacles, indicated by brief descriptive phrases, in hindering private investment in various parts of central cities. Note that for any

