STATE-BASED RECONSTRUCTABILITY MODELING FOR DECISION ANALYSIS

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Reconstructability analysis (RA) is a method for detecting and analyzing the structure of multivariate categorical data. Jones and his colleagues extended the original variable-based formulation of RA to encompass models defined in terms of system states (Jones 1982; Jones 1985; Jones 1985; Jones 1986; Jones 1989). In this paper, we demonstrate that Jones' previous work comprises two separable ideas: the "g to k" transformation and state-based modeling. We relate the concept of state-based modeling to established variable-based RA methods (Klir 1985; Krippendorff 1986), and demonstrate that state-based modeling, when applied to event and decision tree models, is a valuable adjunct to the variable-based sensitivity analyses commonly employed in risk and decision modeling. Examples are provided to illustrate the approach, and issues associated with the interpretation of state-based sensitivity analyses are explored.

Keywords: reconstructability analysis, state-based modeling, decision analysis, k-systems analysis

INTRODUCTION

The focus of this paper is information-theoretic (probabilistic) state-based modeling of directed systems defined by categorical multivariate data. The concept of state-based modeling is inherent in Jones' conception of "k-systems analysis" (Jones 1982; Jones 1985; Jones 1985; Jones 1986; Jones 1989). In the context of k-systems analysis, however, Jones linked the concept of state-based modeling idea to the concept of a "g to k" transformation. We show in this paper that the two concepts can be separated, and that state-based modeling can be viewed as a logical extension of established variable-based RA methods. We also explore the application of state-based modeling concepts to event and decision tree analysis.

In this context, a "system" is what Klir terms a "behavior system" (Klir 1985) -- a contingency table which assigns frequencies or probabilities to system states. A directed system is one in which each variable is distinguished as being either an "independent variable" (IV) or a "dependent variable" (DV). In this paper, the IVs will define the system state and the DVs will depend upon this state. We consider systems with one or more qualitative (categorical or ordinal) IVs and one or more qualitative DVs. Restricting the scope to qualitative variables is not as limiting as it might seem since continuous (and interval- or ratio-scale) variables can be made discrete by clustering ("binning"), although clustering does sacrifice some of the information in the original quantitative variable. Quantitative decision criteria can be readily represented in this framework. In

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fact, decision analyses typically discretize problem variables so they can be represented as nodes in a decision tree representation. There are many possible models for any given system of this sort, and a model's quality is assessed in terms of the degree to which the model accounts for the constraint in the DV and the model's parsimony. We define a model following Krippendorff: "A structural model consists of several components, each specified by a different parameter with respect to which it corresponds to the data to be modeled, and none is included or equivalent to another" (Krippendorff 1986).

STATE-BASED MODELING AND RECONSTRUCTABILITY ANALYSIS

State-based modeling is one aspect of the "k-systems analysis" proposed by Jones (Jones 1986). Jones' method actually encompasses two different concepts, which it is useful to separate. The first concept is that an RA model need not be defined in terms of univariate or multivariate projections of the data, but can instead be defined by specifying the set of system states for which the model frequencies or probabilities are constrained to match those observed in the data. The only requirement is that the constraints imposed on the set of system states must be mutually consistent and linearly independent. The second concept is that any function that maps the system states defined by the IVs onto a finite segment of the real line can be transformed, using what Jones labeled a "g-to-k transformation," into a function with values in the range (0,1) that sum to one. The transformed function can be treated as a probability distribution and subjected to RA. The result, after an inverse transformation, approximates the original function. The term "k-systems analysis" is used by Jones to describe the application of both these concepts to problems of function approximation and data compression. In present research, however, we focus only on the first concept, to which we apply the label "state-based modeling."

In a "state-based" model, the constraint in the DVs is explained in terms of *specific states* of subsets of the IVs. These combinations of IV values (levels) can be viewed as events that are associated with DV outcomes. This perspective is in contrast to the more common "variable-based" modeling perspective, in which constraint in the DVs is explained in terms of *all states* of the independent variables, i.e., the main effects of the individual IVs and interactions among them. State-based modeling is more general than variable-based modeling in that the set of state-based models for a given system contains all possible variable-based models.

We allow the relationship between the IVs and the DVs to be probabilistic, so the system of interest can be defined in terms of a contingency table (joint probability distribution). This represents an important departure from Jones' previous work which considers in effect only one DV (the systems function), which depends deterministically on the system state and is represented, after the g-to-k transformation, in the state probabilities and not as a separate categorical variable. Our approach uses the Jones' state-based modeling idea in a more natural extension of established information-theoretic variable-based RA. This also suggests that the standard statistical methods used to assess model error in variable-based RA (Krippendorff 1986) are applicable to state-based modeling, although we do not attempt to justify this assertion here.

A	**B**	**Z**	count
a0	b0	z0	37
a0	b0	z1	253
a0	b1	z0	195
a0	b1	z1	44
a1	b0	z0	256
a1	b0	z1	277
a1	b1	z0	46
a1	b1	z1	139
	total cases		1247

Table 1. Contingency Table for First Example

The idea that state-based modeling is a natural extension of variable-based RA can be illustrated by means of a simple example. Consider the contingency table shown in Table 1. These data comprise a frequency distribution that summarizes, in the form of counts, observations at each point in the discrete domain defined by the levels of the independent and dependent variables. When normalized with respect to the total number of observations, these counts can be interpreted as a joint probability distribution (Table 2). For instance, in Table 1 if A and B are assumed to be independent variables and Z a dependent variable, each combination of A and B (e.g., "a_0b_1") corresponds to a system state. For every state there is a conditional probability distribution over the levels of Z (z_0 and z_1), also shown in Table 2.

A	**B**	**Z**	Joint PD	Cond PDs
a0	b0	z0	0.030	0.128
a0	b0	z1	0.203	0.872
a0	b1	z0	0.156	0.816
a0	b1	z1	0.035	0.184
a1	b0	z0	0.205	0.480
a1	b0	z1	0.222	0.520
a1	b1	z0	0.037	0.249
a1	b1	z1	0.111	0.751
	total cases		1.000	

Table 2. Joint and Conditional Probability Distributions for Example Data

Information-theoretic variable-based RA can detect and quantify relationships among variables for such contingency tables. RA encompasses both “reconstruction” and “identification” (Klir, 1985). In reconstruction, a distribution is decomposed (compressed, simplified) into component distributions. The ABZ distribution implied by Table 1 might be decomposed into AB and BZ projections, written as the structure, AB:BZ. These two linked bivariate distributions constitute a model of the data in the sense that, through the process of identification, the two distributions can be combined to produce the joint distribution $ABZ_{AB:BZ}$, where the subscript denotes the model from which the joint distribution was generated. This joint distribution associated with a model has the maximum possible information-theoretic uncertainty (Shannon entropy), subject to the constraints imposed by consistency of its component projections with corresponding projections of the data.

The calculated joint distribution associated with the model ($ABZ_{AB:BZ}$) approximates the joint distribution given by the data (ABZ). The quality of the approximation can be assessed with respect to both the information it retains (relative to the data) and its complexity. The information, or constraint, present in the data is the information-theoretic transmission between p, the joint distribution of the data and q_{model}, the joint distribution of the model, where transmission is defined as

$$T = \sum p \log (p / q_{model}) \qquad [1]$$

For the directed system represented by Table 1, we are interested in the effects (individual and joint) of the independent variables A and B on the dependent variable Z, and not in the relationship (if any) between A and B. Therefore, the appropriate reference model is AB:Z since this model assumes that the probability distribution over the states of Z is independent of both the variables A and B. When the data in Table 1 is viewed as a directed system, the "independence" model AB:Z retains none of the information contained in the data about the dependence of Z on the two IVs. Using this independence model as a baseline, we can describe any other possible model (e.g., AB:BZ) in terms of the percentage of the information in the data that is retained by the model. This use of the independence model as a baseline represents another departure from the previous work of Jones, who considered the uniform distribution the appropriate baseline when assessing the information retained by an approximation to a function.

The complexity of a model in variable-based RA is defined as its degrees of freedom (df) i.e., the number of parameters needed to specify the model. For example, seven df are associated with the data shown in Table 1. The model AB:BZ can be shown to utilize five df, after accounting for the fact that the variable B appears in both component projections. The least complex model, AB:Z, postulates that neither A nor B can explain any of the variability in Z and utilizes four df.

Table 3 summarizes the results obtained when variable-based RA is applied to the data from Table 1. For each candidate model, the results include the transmission (relative to the reference model AB:Z), a likelihood ratio Chi-square statistic (L^2 in the table), and the df for the model. From these results, the probability of a Type I error (*p*) can be derived. The null hypothesis in this framework is that the model fits the data, and p is the probability of making an error if we reject the null hypothesis. Therefore, in contrast to the more common orientation in hypothesis testing, a large value of p is desirable in this setting. The last column in the table is the percent of the information in the data that the model retains, calculated as $1 - (T_{model}/T_{AB:Z})$.

Model	T	L^2	df	p	%I
ABZ	- - -	0.00	7	1.0000	100%
AB:AZ:BZ	0.14780	255.51	6	0.0000	17%
AB:BZ	0.14823	256.25	5	0.0000	17%
AB:AZ	0.17773	307.25	5	0.0000	0%
AB:Z	0.17796	307.65	4	0.0000	0%

Table 3. Variable-Based Reconstructability Analysis Results

The results shown in Table 3 indicate that, for this example, all models simpler than the data itself generate frequencies that are clearly inconsistent with the observed frequencies (p = 0.000). Conceptually, this means that the conditional distributions for the dependent variable Z differ significantly across the states defined by the independent variables A and B. When the granularity of the analysis stops at the level of variables, no model simpler than the data appears to explain adequately the observed frequencies. With respect to both explanatory power and parsimony, the best variable-based model simpler than the data itself is AB:BZ which asserts that the variation in the conditional distributions for Z can be explained solely in terms of a single independent variable, B. This model, however, captures only about 17% of the information in the data.

Model	T	L^2	df	p	%I
ABZ	- - -	0.00	7	1.0000	100%
AB:Z:a_0BZ	0.00016	0.27	6	0.6029	100%
AB:Z:a_0b_1Z	0.06960	120.32	5	0.0000	61%
AB:Z:a_0b_0Z	0.08758	151.40	5	0.0000	51%
AB:Z:a_1b_1Z	0.16101	278.35	5	0.0000	10%
AB:Z:a_1b_0Z	0.17201	297.35	5	0.0000	3%
AB:Z	0.17796	307.65	4	0.0000	0%

Table 4. State-Based Reconstructability Analysis Results

A state-based modeling approach can do significantly better in this particular example, as indicated by the results in Table 4. The notation used to describe state-based models is somewhat more complex and requires explanation. Since the AB:Z independence model is the reference model for this directed system, all state-based models must include both the AB and Z components. This forces those states which are not explicitly constrained in the state-based model to be maximally consistent with AB:Z.

To obtain the joint probability distribution associated with the model AB:Z:a_1b_0Z, iterative proportional fitting (Bishop, Fienberg et al. 1975) is used to maximize the information-theoretic uncertainty of the joint distribution, subject to the imposed constraints. There is one set of constraint equations associated with each component in the model. In this case, AB and Z projections of the model distribution must be consistent with the corresponding marginal projections observed in the data, and the distribution over Z conditioned on the a_1b_0 state is also constrained to match the observed conditional distribution for that state. This relaxation procedure has the effect that, in the model, the conditional distributions on Z for all states other than a_1b_0 are as similar as possible, subject to satisfaction of the AB and Z constraints. This is illustrated in Table 5 for four different state-based models, including the AB:Z:a_0b_1Z model.

	data		AB:Z:a0b0Z		AB:Z:a0b1Z		AB:Z:a1b0Z		AB:Z:a1b1Z	
	p(z0)	p(z1)	p(z0)	p(z1)	p(z0)	p(z1)	p(z0)	p(z1)	p(z0)	p(z1)
a0b0	0.128	0.872	0.128	0.872	0.336	0.664	0.389	0.611	0.459	0.541
a0b1	0.816	0.184	0.519	0.481	0.816	0.184	0.389	0.611	0.459	0.541
a1b0	0.480	0.520	0.519	0.481	0.336	0.664	0.480	0.520	0.459	0.541
a1b1	0.249	0.751	0.519	0.481	0.336	0.664	0.389	0.611	0.249	0.751

Table 5. Marginal Z Distributions for State-Based Models

In practical terms, how do we interpret such a model? Suppose the model AB:Z:a_0b_1Z is found to be very informative. In that case, if we want to predict the behavior of the dependent variable Z, what we most need to know is whether or not the system is (or will be) in the state a_0b_1. There is one distribution over the states of Z conditioned on the system being in the state a_0b_1. A second "default" conditional distribution over the states of Z can be used for any other combination of A and B, without significantly compromising the quality of our prediction.

As shown in Table 4, the state-based models AB:Z:a_0b_0Z and AB:Z:a_0b_1Z each do much better, in terms of information captured, than the best variable-based model. Because the sample size (n=1247) is fairly large, however, the discrepancies between the observed and predicted frequencies are sufficient to keep p low and justify a statistical rejection of both models. The combined model AB:Z:a_0BZ, however, captures virtually all of the information in the data (I = 100%), is still less complex than the data itself (df = 6), and has p = 0.6 (i.e., there's a substantial probability that we'd be wrong if we rejected this model based on the small discrepancies between the observed and predicted frequencies). Again, the AB:Z:a_0BZ model can be best interpreted in terms of a "strategy" for explaining or predicting Z. This model says that to predict Z, we need to know whether or not the system is in the a_0 state. If so, then there is one conditional distribution for Z if B = b_0, and a second conditional distribution if B = b_1. A third conditional distribution for Z is applicable whenever A = a_1, regardless of the value of B. This model is simpler than the data; its specification utilizes one less degree of freedom. It will do an excellent job predicting the value of Z, however, given information about variables A and B.

Although it is not relevant to the present example, those states that are explicitly constrained in a model need not always include the entire Z distribution. When Z is dichotomous, as in the present example, then the model AB:Z:a_0b_1Z is effectively the same as the model AB:Z:$a_0b_1z_0$ since the conditional Z distribution must sum to one. However, when Z is a multichotomous variable, then it may be advantageous to constrain only some of the probabilities in a conditional Z distribution to match the data; the rest of the conditional distribution will be adjusted to be maximally consistent with the overall marginal Z distribution. Such a model still yields a conditional Z distribution for every state defined by the independent variables, and the interpretation of the model is as described above. The model may be more efficient, however than one which forces consistency with the entire observed conditional Z distribution for all constrained states.

This simple example illustrates the fact that state-based modeling is a natural extension of information-theoretic, variable-based RA. The benefits of this approach, however, are most apparent when state-based modeling is applied to data sets involving many variables linked through complex higher order interactions. In such situations, these techniques may provide valuable insight and guidance to an analyst, and serve as an important adjunct to more established analytical tools.

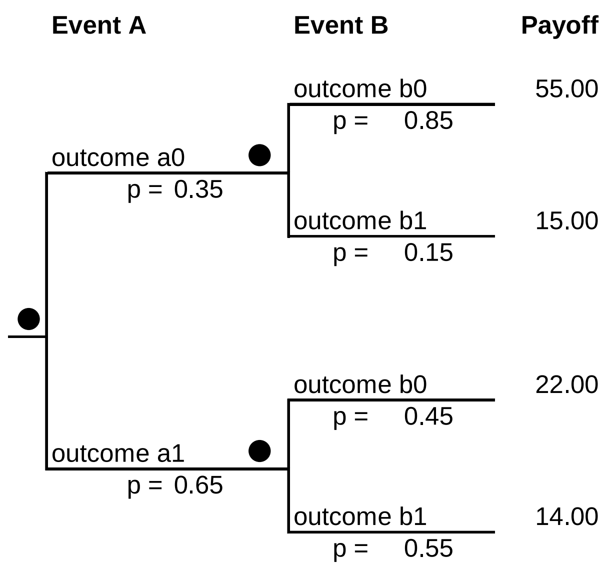
STATE-BASED MODELING AND DECISION ANALYSIS

We turn now to a second example, one that explores the application of state-based modeling to the analysis of event and decision trees. This example, which is *non-statistical* in nature, will demonstrate that an event tree in which each path has an associated utility for a decision maker is a special case of the qualitative multivariate categorical distribution analyzed the first example above. From this perspective, state-based analysis provides a new and potentially powerful alternative to the standard variable-based sensitivity analysis methods commonly employed by decision analysts.

To set the stage for this example, a brief discussion of decision analysis and sensitivity analysis will be helpful. Decision analysis (Raiffa 1968; Howard 1988; Clemen 1996) can be viewed as one discipline within a spectrum of decision-aiding technologies (Howard 1992). Decision analysis is characterized by its normative orientation, its axiomatic foundation, and its use of mathematical models to characterize decision situations. In general, the models developed by decision analysts produce two kinds of results. The first result is a policy recommendation that will maximize expected utility, given the decision alternatives available and the uncertain events that are explicitly characterized in the model. This is often referred to as the “solution” of the decision model. The second result is insight about how sensitive this solution is to the assumptions included in the decision model. In practice, the second result is at least as valuable as the first.

The model assumptions tested by sensitivity analyses include the specific outcomes for the uncertain events in the model, the probabilities associated with those outcomes, and the values for deterministic parameters included in the model. A sensitivity analysis is conducted by solving the decision model at a number of discrete steps across the range of possible values for a model assumption, recording at each step the recommended policy and the decision maker’s resulting utility. These results provide insight into the potential benefit of improving the decision maker’s information about or control over the ultimate realization of the model assumption being analyzed.

Generally, this process is carried out for one variable at a time. An ambitious analyst, suspecting an interaction between two assumptions in their effect on the decision maker’s utility, may conduct sensitivity analysis in which two-variables are considered simultaneously. Specialized graphical display formats have been developed to summarize the results of both one- and two-variable sensitivity analyses (Eschenbach 1992). Analyses involving more than two variables are rarely carried out, however, and three-variable sensitivity analysis is supported in only one of the computational packages available commercially at this time (Palisade 1997; ADA 1999; TreeAge 1999). Problem formulations involving large numbers of uncertain variables are becoming more common, however, as computer hardware becomes more capable and decision analysis software becomes more sophisticated and widely available. In such applications, the ability to conduct more complex sensitivity analyses would be extremely valuable. State-based modeling may provide a means to address this need.



Probabilities

	b0	b1
a0	0.298	0.053
a1	0.293	0.358

Payoffs

	b0	b1
a0	55.00	15.00
a1	22.00	14.00

Figure 1. Event Tree with Payoffs

As a first step towards this end, consider the very simple event tree[1] shown in Figure 1. Such an event tree might be found subsequent to a branch of a decision node in some larger decision tree. Each path through the tree represents a possible outcome for the decision-maker, presumably based on some previous choice of an alternative. At each node in the tree, the probability associated with each branch is shown immediately below the branch. Associated with each path through the tree is a payoff to the decision maker, as shown in the figure. These payoffs could be in any appropriate units, but we assume that at this point, they have not been converted to utilities. Assuming that it is consistent with the axioms governing utility theory (Clemen 1996) and that it reflects the risk preferences of the decision maker, the conversion from payoff to utility is somewhat arbitrary. For the purpose of this example, we assume that the decision maker has assigned a utility of zero to a payoff of -100 and a utility of one to a payoff of 100. We also assume that within this range of payoffs (i.e., the interval [-100,100]) the decision maker's utility function is linear, so she will value both alternatives and intermediate

[1] An "event tree" is a tree that has only probability (i.e., event) nodes and no decision nodes.

outcomes on the basis of their expected payoff in a standard gamble[2]. Figure 2 shows the event tree under consideration with the resulting utilities substituted for the original payoffs. (The payoff of 55, for example, becomes converted to a utility = [55 - (-100)]/[100 - (-100)] = .775.)

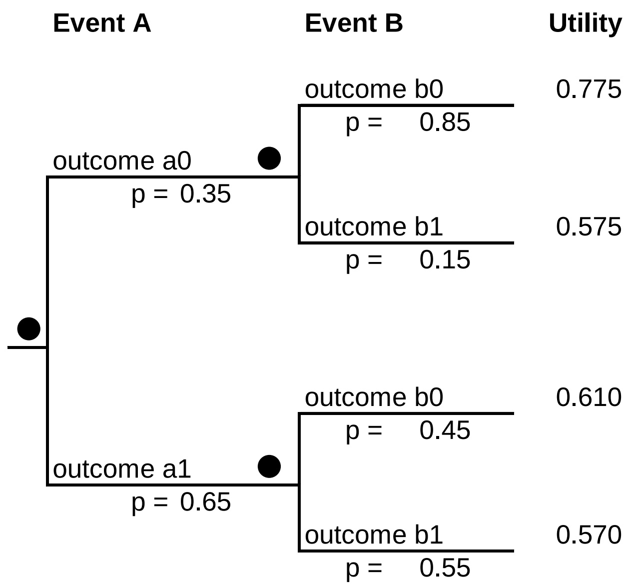


Figure 2. Event Tree with Utilities

By the substitutability axiom of utility theory, the decision maker is indifferent between a utility of 0.775 and an uncertain event that has the same expected utility. For our purposes, a convenient event is the lottery that yields the maximum utility of one with probability 0.775 and the minimum utility of zero with probability 0.225 (since 1.000 - 0.775 = 0.225). If we make this substitution for all the utilities and apply the labels z_0 and z_1 to events with utilities zero and one respectively, then we can transform our original event tree into the one shown in Figure 3. The figure also shows the implied joint probability distribution for the variables A, B, and Z to the right of the tree.

With these changes, we are on the familiar ground of our first example. We have transformed the event tree and its associated payoffs into a multivariate probability distribution defined over a set of qualitative variables. The state-based analysis method described in the first example can be directly applied to this transformed tree. We wish to stress, however, that our integration of utility considerations into the framework of reconstructability analysis is a more general contribution, which can be employed within standard variable-based RA.

[2] Linearity is not required here; any valid utility function could be employed at this point in the process.

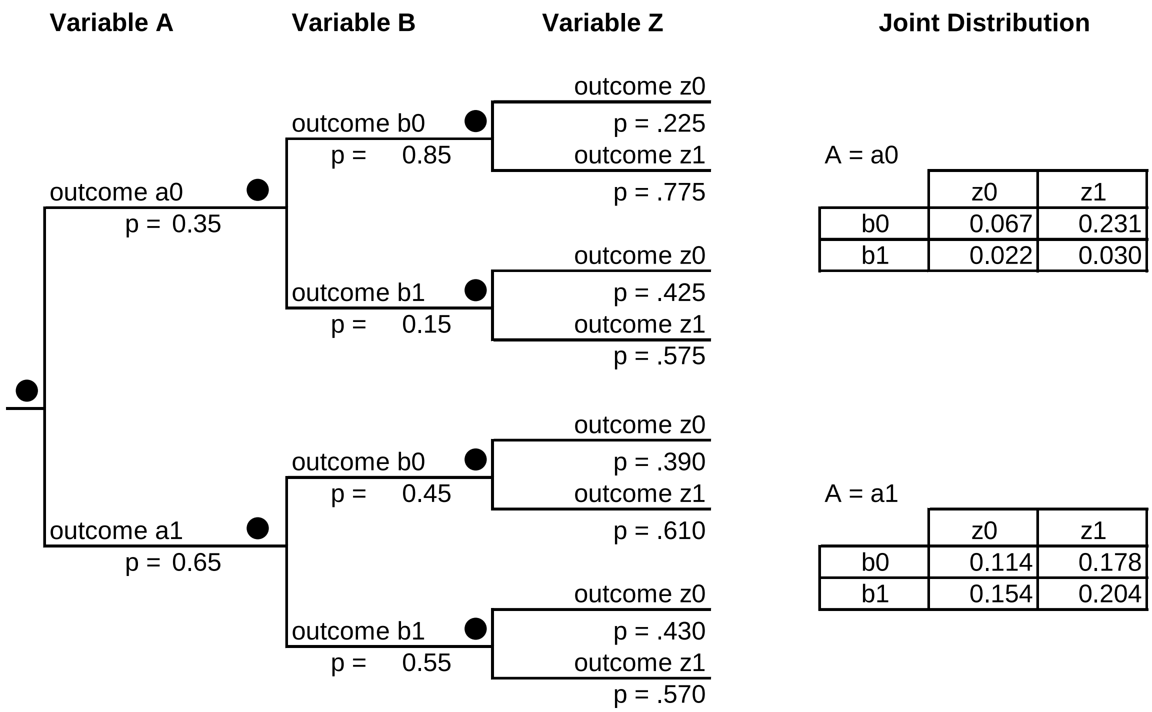


Figure 3. Expanded Event Tree Ready for State-Based Analysis

Recall that what the decision maker seeks from a sensitivity analysis is insight into which outcomes or combinations of outcomes will lead to an especially favorable or unfavorable utility. Another way to state this is that the decision maker wants to know which outcomes or combinations of outcomes are most responsible for the variability in the utility she realizes. This is exactly the information that a state-based analysis will provide when we analyze the tree in Figure 3 as a directed system with Z as the DV.

Model	T	df	%I
ABZ	- - -	7	100%
AB:Z:a_0b_0Z	0.00079	5	97%
AB:AZ:BZ	0.00340	6	86%
AB:AZ	0.00698	5	72%
AB:BZ	0.01374	5	45%
AB:Z:a_1b_1Z	0.01588	5	37%
AB:Z:a_1b_0Z	0.02369	5	6%
AB:Z:a_0b_1Z	0.02431	5	3%
AB:Z	0.02509	4	0%

Table 6. Event Tree Analysis Results

The results of such an analysis are shown in Table 6. These results could be communicated to a decision maker by stating that "The excellent fit of model AB:Z:a_0b_0Z suggests that a very large percentage of the variability in the utility you will realize from this situation is associated with the occurrence (or non-occurrence) of event a_0b_0. Since this event results in a high payoff for you (55.00, as indicated in Figure 1), you should think hard about how you could make it more likely that this event will occur, or improve your ability to forecast whether or not it will occur." (If, instead, the event a_0b_0 had very low utility, the decision might want to seek insurance that limited her losses if the event a_0b_0 did occur.) Using the AB:Z:a_0b_0Z model as an approximation reduces Figure 3 to the simpler tree in Figure 4.

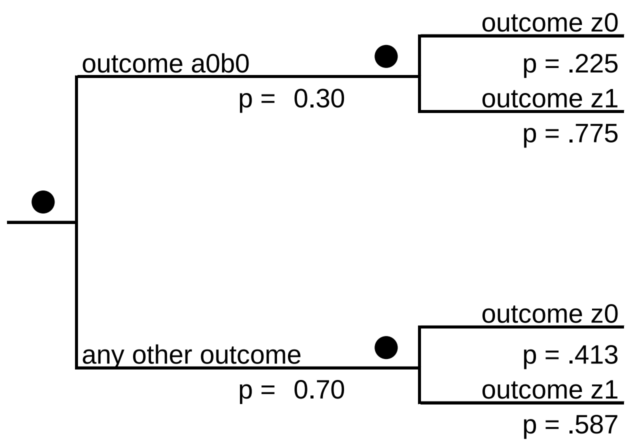


Figure 4. Simplified Event Tree Based on Model AB:Z:a_0b_0Z

In this very simple example, a quick inspection of the original event tree shown in Figure 1 would have led immediately to similar conclusions. In a realistic decision problem involving dozens of relevant uncertainties, however, an insight of this sort can be extremely difficult to obtain, especially when the outcomes that jointly define the "important" event are associated with variables that are scattered throughout a large event tree. To some extent, insight can be gained by examining the terminal branches of the tree, looking for paths that result in very high or very low utility. However, this *ad hoc* approach often fails to illuminate situations in which the variance in the utility is driven by events that are resolved early in the tree (i.e., far to the left). In these cases, the resulting benefit or harm is diffused across the multiple paths created by later nodes in the tree, and the effect is not easily discerned by inspection of the terminal branches. The difficulty is compounded when the marginal probability distribution for one or more of the variables involved is conditional on a large number of other variables. In such situations, existing approaches to sensitivity analysis are cumbersome at best and often simply impractical. While further development and validation of state-based modeling is obviously required, the approach may prove to be a valuable addition to the decision analyst's toolbox in such situations. By providing an alternate perspective on the structure of an event tree, information-theoretic state-based analysis may help focus the application of more established techniques. The technique may be especially valuable for very large trees involving many variables and complex probabilistic dependencies.

CONCLUSIONS

State-based modeling is a powerful and broadly applicable generalization of established, variable-based reconstructability analysis techniques. The best state-based models for a given data set will always retain at least as much information as the best variable-based models, with no increase in model complexity. In many cases, however, state-based models will retain more information while simultaneously reducing model complexity. In this paper, we alter Jones' state-based modeling approach and integrate it more naturally into RA by

(a) allowing more than one DV and treating the DVs as ordinary RA variables which can depend probabilistically on the IV,
(b) choosing as the base reference model for directed systems the independence model and not the uniform distribution, and
(c) adding statistical assessment of the state-based models.

A promising application of state-based modeling is the simplification of event and decision tree models. The traditional sensitivity analysis techniques used as a basis for model simplification are easily overwhelmed when problems involve a large number of variables. By highlighting the specific events and combinations of events that drive the variability in a decision maker's ultimate payoffs, state-based modeling can provide valuable and actionable insights into the structure of complex decision and risk analysis problems. Moreover, the innovation we introduce of encompassing utility considerations within the RA framework is a general one, which can be used also in standard variable-based modeling.

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