Capital tax competition and social security

Abstract The classic capital tax policy externality is studied in the presence of a social security program where both the benefits and taxes depend on wages in an overlapping generations economy with many countries and mobile capital. We study the response and welfare implications of a coordinated capital tax rate increase across countries competing for the mobile tax base on the initial generations, the transition, and the steady state. The tax increase is initially completely capitalized but some of the burden is shifted to labor on the transition path and in the steady state. Several new welfare effects are uncovered including an effect involving the parameters of the social security program. Sufficient conditions are provided so that all current and future generations are better off from the reform. However, social security may reduce the gain to capital tax reform.

Keywords *Tax competition · social security · overlapping generations · savers and myopes*

JEL Classification H2 · H5 · H7 · R1 · R5

1 Introduction

The purpose of this paper is to extend the literature on horizontal capital tax competition across countries by including a pay-as-you-go social security program that depends on wages, in addition to a source based tax on capital in the context of an overlapping generations economy.[1] If capital tax competition effectively keeps the tax on mobile capital low, the amount of capital invested in the economy will be high, and wages will be high as a result. This will bring in extra revenue into the social security program through the payroll tax and make it easier to support the program. However, it may also cause a drain on the cash flow of the program over time if benefits are also tied to past wages. A coordinated reform across countries that raises the capital tax rate in order to alleviate the capital tax policy externality will generally affect local wages and will interact with the social security program. For example, if the wage falls when the reform is introduced, this will reduce the amount of payroll tax revenue flowing into the system and may call for a benefit cut, a payroll tax increase, or both to maintain the budget of the program.

Most countries have social security programs, and in many of them, a worker's contribution is based on a payroll tax imposed on labor earnings. In addition, the benefit received from the program is generally tied in some way to the worker's earnings history prior to retirement by the benefit replacement rate. Many programs also contain lump sum elements in the benefit that are not tied to a worker's wage history. For example, some programs pay a universal benefit that is intended to finance a basic minimum of retirement consumption for low wage workers, disabled workers, or individuals who do not have a significant work history in the formal labor market. Finally, in many countries general tax policy and social security policy are debated, chosen, and budgeted completely separate from one another. We incorporate these features into our model.[2]

We focus on a reform where all countries permanently increase their capital tax rate in order to alleviate the capital tax policy externality and study the positive and normative effects of the reform. We show that a permanent increase in all local capital tax rates is completely capitalized in the interest rate in the first period of the reform. The local wage and the payroll tax rate do not respond in the first period as a result. However, savings responds to the capital tax reform and complete capitalization does not occur after the first period. The interest rate and the wage both continue to fall on the transition path and in the steady state. This puts downward pressure on both payroll tax revenues and eventually on tied benefits when both depend on the wage. We show that payroll tax revenues begin to fall in the second period of the reform since current payroll tax revenue depends on the current wage. The tied benefit in the second period of the reform, however, depends on the lagged wage and is unaffected until the third period of the reform. In a sense, this creates an endogenous funding crisis for the social security program.

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[1] See Zodrow and Mieszkowski (1986), Wilson (1986), Mintz and Tulkens (1986), and Wildasin (1988, 1989) for the early research. See Wilson (1999) and Wilson and Wildasin (2004) for surveys of the literature. One classic result is that competition among governments for a mobile tax base like capital may lead to such a tax being set too low, which would put downward pressure on public spending. Another classic result is that a coordinated increase in the capital tax rate across all governments competing for the base would increase public spending and improve welfare.

[2] See Social Security Programs Throughout the World at www.ssa.gov for a description of the features of social security programs by country.

There are a number of new welfare effects to consider including the spending effect in the current period when the agent is young, a spending effect in the future when the agent is old, a wage effect, an interest rate effect, and the impact of the reform on the social security program, which works through the wage. We also provide sufficient conditions such that every generation benefits from the reform. However, we also show that the existence of social security based on wages makes it harder (easier) for the reform to improve welfare when the present value of payroll tax revenue is greater (lower) than the present value of tied benefits at the margin.

There is a related literature that studies reforms of social security in dynamic models where economies are connected by a capital market and capital is mobile. Pemberton (1999, 2000) argues that capital accumulation will increase as countries shift to a jointly coordinated pension program. However, Beltrametti and Bonatti (2003) show that saving and capital accumulation may decrease if the current government takes into account the lifetime welfare of generations currently alive when it reforms the public pension program. Casarico (2000) shows that capital investment will be lower and the local interest rate higher under a pay-as-you-go program than if the program is fully financed. It follows that a country will benefit more when capital is mobile under pay-as-you-go financing. Pestieau, Piaser, and Sato (2006) study the tradeoff between redistribution across generations and redistribution within a generation. They show there is more intragenerational redistribution but less capital accumulation as the number of countries in the economy grows.

A main theme running through this work is that if social security taxes and transfers generally reduce saving in each country and each individual country does not take this into account, then world capital markets will be affected by this. So consider one country's decision to increase its social security program. If this reduces savings in that country and this spills over to other countries, then it will reduce wages in other countries. Since this negative externality is not taken into account in the first country, it will tend to choose a program that is too large. If this is generally true, then social security programs will be inefficiently large.

This line of research studies social security programs that rely on a payroll tax under pay-as-you-go financing. However, the benefit is not tied to wages. Capital taxes are not included. It also does not consider the horizontal policy externality problem of the capital tax, which is the focus of our attention. Finally, previous work in this area has not provided a reason for social security to be included in the model. If social security improves welfare locally within the country because it addresses a domestic market failure, or has a favorable effect on the local income distribution, then coordinating the program across countries may not make sense if it means that social security can no longer address the domestic problem it was designed for. We assume each generation has life cycle savers and myopic agents who do not save and label them spenders, following Feldstein (1985) and Mankiw (2000). Social security can then be justified as a forced saving program and it may also have a favorable effect on the distribution of welfare across agents within the country.

Feldstein assumes the economy is closed, factor prices are fixed, there is only one policy under study, namely, social security, and only examines the steady state. Payroll taxes imposed on the wage finance the program and the benefit is not tied to the wage. He also ignores the impact of the policy on the initial generations. We extend this to an overlapping generations economy where there are many countries, each country has a general tax policy in addition to social security, there is a source-based tax on capital, factor prices are endogenous, and benefits are tied to the wage. We also consider the impact on the initial generations alive at the beginning of the reform, those alive in the transition, and in the steady state.[3]

In the next section, we discuss some of the general features of social security programs worldwide to provide some background for our assumptions. We present our model in section 3. In section 4 we present and discuss the optimal policy rules. In section 5 we study the positive effects of a coordinated increase in all capital tax rates on the initial equilibrium. The effects of the coordinated reform on welfare are discussed in section 6. Section 7 provides an example where the welfare effects are mixed; some are better off, e.g., spenders, while some are not, e.g., the initial old savers. Section 8 concludes the paper. The results are derived in the Appendix.

2 Common features of social security programs

There is a great deal of diversity in the provisions of social security programs worldwide.[4] Many of the specific details depend on local political concerns and problems that are literally unique to the country itself. However, there are also a number of provisions that are common across programs.

In many countries the social security program's budget is considered completely separate from the government's general budget. In addition, when changes are made to one program, the other is not usually considered. For example, in the early 1980s reforms to the social security program were undertaken in the US without discussion of the funding for general government spending, and the social security program was ignored when the government introduced the 1986 income tax reform.

Social security programs typically use a payroll tax to finance benefits and many operate on a pay-as-you-go basis. The tax is typically imposed on both the firm and the worker. However, most analysts believe that workers bear most of the burden of the tax. This also follows much of the literature which assumes the tax is imposed only on the worker. There is a great deal of variation in the payroll tax rate. The combined tax rate for the basic old age, disability, and survivor program in 2004 - 2005 for some countries tends to be low, e.g., 9.9% in Canada, 12.4% in the United States, and 13.58% in Japan, but much higher in other countries, 19.5% in Germany, 23.8% in the UK, and 32.7% in Italy.

Programs also typically pay a benefit that is tied to the retiree's own earnings history, and hence past wages, by the so-called replacement rate. It is also quite common to use only a fraction of the worker's earnings history. The replacement rates tend to be high in Europe, e.g., 75% in Sweden, 63% in Germany, and 61% in France, although, some countries have lower replacement rates, e.g., 35% in the UK. Many countries also index the benefit for inflation.

Many programs also contain lump sum elements associated with the benefit that do not depend on the retiree's earnings history. For example, many programs pay a so-called universal benefit as part of the monthly transfer. Indeed, many developed countries, e.g., Denmark, Norway, Russia, the UK, Switzerland, Japan, and Canada,

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3 Cremer, De Donder, Maldonado, and Pestieau (2008) also use a version of the Feldstein model to study social security. They examine the program in a closed, partial equilibrium economy where the interest rate and the growth rate are both fixed at zero, and there is a fixed distribution of wages based on productivity for each type of agent. Their focus is on the relationship between the number of myopic agents and the structure and size of the program. They also do not address policy externality issues.

4 See Social Security Programs Throughout the World at www.ssa.gov.

pay both an earnings related benefit and a universal pension. This type of benefit is typically paid to ensure there is a minimum level of consumption in retirement, especially for those who do not have a significant work history.[5]

Most developing countries also have social security programs. They typically rely on a payroll tax, are pay-as-you-go, are separate from the general budget, and pay a benefit tied to the retiree's past earnings. However, tax rates and replacement rates tend to be lower than for developing countries. In addition, a few developing countries also pay a universal benefit, e.g., Bolivia, Brunei, Botswana, Mauritius, and New Zealand, and some developing countries also have other lump sum elements like a fixed survivor benefit as well.

There are many reasons that can justify the existence of a social security program, e.g., potential risks that cannot be insured against like an unexpected downturn in the stock market. One paternalistic reason for such a program is that myopic individuals may not fully understand the need to save for their own retirement, or do not have the necessary willpower to do so. There is strong evidence that a significant portion of the US population does not follow the life cycle model.[6] Pay-as-you-go social security can be justified as a forced saving program to ensure that everyone contributes to the system, especially myopic agents. Following Feldstein (1985), Mankiw (2000), and, more recently, Cremer, De Donder, Maldonado, and Pestieau (2008), we will assume the population is made up of a mix of life cycle savers and myopic agents. We will refer to the latter group as spenders. Following this literature, we will assume spenders are myopic because they have inconsistent preferences and recognize the need to save for retirement only after retirement has occurred.[7]

3 The analytical framework

3.1 Basic structure

Time is discrete and the economy lasts forever, t = 1, 2, ... There are J > 1 identical countries. In each country there are N_t young agents born at time t and population grows at rate n. Each person lives for two periods, young and old, each agent is endowed with one unit of labor when young only, and the endowment is completely supplied to the labor market in exchange for a wage. There are two types of agent within each generation, savers, who follow a life cycle pattern of saving, and

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5 Some benefits may be lump sum relative to the person receiving the transfer even if they are determined by the earnings of the main beneficiary. In many programs a survivor may receive a benefit after the main beneficiary passes away. This transfer would be lump sum relative to the survivor. As another example, some programs pay a benefit to the spouse of the main beneficiary that is dependent on the latter's earnings history, not the recipient's, e.g., US. And, finally, individuals who are mentally challenged or disabled early in life and unable to work in the US do not have a work history and are not eligible for the disability benefit. Instead, they receive a payment from the Supplemental Security Program, which is clearly lump sum in nature.

6 For example, see the evidence in Thaler (1981, 1990), Campbell and Mankiw (1990, 1991), and Mankiw (2000).

7 This is related to the research on consumer myopia and the hyperbolic discounting literature. For the early research on models of consumer myopia and dynamic inconsistency see Strotz (1956) and Phelps and Pollack (1968). For empirical evidence of myopia see Thaler (1981, 1990) and Campbell and Mankiw (1990, 1991), for example. And for research on hyperbolic discounting see Laibson (1997) and Harris and Laibson (2001).

spenders, who do not save at all. The agent's type is private information until the beginning of the second period of life when their savings behavior is revealed. Let ϕ denote the fraction of savers and $1 - \phi$ the fraction of spenders, where $0 < \phi \leq 1$. Agents within a type are identical within a generation and across generations.

Capital is perfectly mobile across countries, while labor and people are immobile. There is one private output available that is produced by competitive firms according to a well-behaved, constant returns to scale technology using capital and labor. Output of the private good per worker in the representative country at time t is given by $y_t = f(k_t)$, where k_t is capital per worker at time t. The technology satisfies the usual assumptions.

Following the discussion of the last section, each country has two agencies that are separate and distinct from one another, a general tax authority (GTA) and a social security authority (SSA). The GTA imposes a source-based tax on mobile capital to finance a public good, while the SSA operates a pay-as-you-go social security program, and the agencies act independently of one another. Both the young and the old benefit from the public good provided by the GTA. The private good can be converted into the public good on a one-for-one basis and it does not confer any spillover effects. Social security pays a benefit that depends on the worker's earnings history and a universal lump sum benefit and is financed by a payroll tax on the wage.

3.2 Sequence of action

At time t = 0 each government agency in each country chooses the entire time path for its respective policy variables, $\{\tau_t, g_t\}_t$ for the GTA and $\{\gamma_t, \beta_t\}_t$ for the SSA, simultaneously, where a location subscript has been omitted, $\{x_t\}_t$ denotes an infinite sequence, and where τ is the capital tax rate, g is a public good, γ is the payroll tax rate, and β is the benefit replacement rate. Each agency takes the policies of the other agency, other countries' policies, and aggregate variables like the world interest rate as given, but takes into account the impact of its policy on local wages. Agencies are thus not choosing policy strategically vis-a-vis other agencies. The policy rules are implemented at t = 0 and fixed. Each period the old generation of savers observes the entire set of policies across countries and allocates its capital across countries. It then combines its capital with labor to produce the output of the private good. Factors receive their respective factor payments, taxes are paid to finance the public good, and taxes and transfers are paid through the social security system. Consumption occurs and some of the income of the young savers is saved for next period when they become the capital owners, as in the classic OG model.

3.3 Firms

Old savers own the firms and each firm behaves competitively by taking the actions of all other agents as given. Since firms are identical, the technology exhibits constant returns, and firms behave competitively, we will normalize on firms so there is a measure of magnitude one of firms. The firms use labor and capital to produce the private good. Profit per worker for a firm is $f(k_t) - (r_t + \tau_t)k_t - w_t$, where r_t is the real interest rate determined in the world capital market, τ_t is the source based capital tax rate, w_t is the local wage per worker, and a location subscript has been omitted for brevity.

The firm chooses capital per worker to maximize profit. Thus,

$$df/dk_t = f_k(k_t) = r_t + \tau_t = r_{nt}, \quad (1)$$

where f_k is the marginal product of capital per worker in the representative country and r_{nt} is the net user cost of capital. We can solve equation (1) to obtain the capital demand function per worker, $k_t = K(r_{nt})$, where the derivative is $dk_t/dr_{nt} = dk_t/d\tau_t = K_r = 1/f_{kk} < 0$, and $f_{kk} < 0$ is a second derivative. Constant returns to scale implies that economic profit is zero. Hence, the wage function is given by

$$w_t = f(K(r_{nt})) - r_{nt}K(r_{nt}) = W(r_{nt}), \quad (2)$$

and the response is given by, $W_r = dw_t/dr_{nt} = dw_t/d\tau_t = -k_t$ by the envelope theorem.

3.4 Consumers

We will rely on a life cycle model of consumer behavior for savers.[8] Preferences are represented by a well-behaved utility function of the general form, $U^t(c_{1t}, c_{2t+1}, g_t, g_{t+1})$, where c_{1t} is consumption in the first period of life at time t, c_{2t+1} is second period consumption that occurs at time t+1, and g_t is the public good available at time t. Utility is additively separable for simplicity, and for the generation born at time t it is given by, $U^t(c_{1t}, c_{2t+1}, g_t, g_{t+1}) = u^t(c_{1t}) + v^t(g_t) + b[(u^t(c_{2t+1}) + v^t(g_{t+1})]$, where b is the discount rate. Hence, savings is not a function of the public good, a standard assumption. We will use subscripts to denote partial derivatives, $\partial U^t/\partial c_{1t} = U^t_1 = u_1(c_{1t})$, $\partial U^t/\partial c_{2t+1} = U^t_2 = bu^t_1(c_{2t+1})$, $\partial U^t/\partial g_t = U^t_3 = v^t_1(g_t)$, and so on.

We will use an asterisk to denote the variables of the spender. Following Feldstein, Mankiw, and Cremer, et. al., ex ante, the spender does not recognize the need to save and hence their utility function in their first period is $u^{*t}(c^*_{1t}) + v^{*t}(g_t)$. However, ex post, they regret not having saved for retirement and their utility is $U^{*t}(c^*_{1t}, c^*_{2t+1}, g_t, g_{t+1}) = u^{*t}(c^*_{1t}) + v^{*t}(g_t) + b^*[u^{*t}(c^*_{2t+1}) + v^{*t}(g_{t+1})]$ instead. This inconsistency provides a reason for the existence of social security in order to force spenders to save.

Following the earlier discussion, consumers confront the following social security parameters. A social security tax of $\gamma_t w_t$ is paid into the system when young at time t. The benefit received from the program includes a universal transfer of α_{t+1} and a transfer that depends on the individual's own earnings history, $\beta_{t+1}w_t$, where $0 < \beta < 1$. The universal benefit will be taken as exogenous throughout. The saver's budget constraints are

$$(1 - \gamma_t)w_t - c_{1t} - s_t = 0,$$
$$(1+r_{t+1})s_t + \alpha_{t+1} + \beta_{t+1}w_t - c_{2t+1} = 0,$$

where s is retirement savings. The budget constraints of the spender are the same as the saver except with $s^* = 0$. We will also assume agents cannot borrow against their future social security benefit, which seems realistic.

The saver chooses consumption and savings to maximize utility subject to the budget constraints. The first order condition is $U_1/U_2 = u_1(c_1)/bu_1(c_2) = (1+r)$. The solution to the decision problem is a savings function of the following form, $s_t =$

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[8] We have not included cash bequests. Bequeathing behavior is very complicated and there are many motives for such transfers. Changes in intra-family transfers will not offset changes in the local capital tax policy under any motive including pure altruism since that policy causes distortions across firms, not consumers. In addition, there are theoretical objections and considerable empirical evidence against the pure altruism model. See the discussion in chapter 11 in Batina and Ihori (2000).

$S(w_t(1 - \gamma_t), \alpha_{t+1} + \beta_{t+1}w_t, r_{t+1})$, where $I_{1t} = w_t(1 - \gamma_t)$ is first period disposable income and $I_{2t+1} = \alpha_{t+1} + \beta_{t+1}w_t$ is social security income received in the second period of life. If consumption in both periods is a normal good, then savings is increasing in first period income and decreasing in second period income.[9] If the substitution effect dominates, then savings will be increasing in the interest rate. Thus, $\partial s/\partial I_1 = S_1 > 0$, $\partial s/\partial I_2 = S_2 < 0$, and $\partial s/\partial r = S_r > 0$, respectively. It can also be shown that $S_2 = - R(1 - S_1)$, where $R = 1/(1+r)$. Finally, the response of saving to the wage has two components. The wage affects income directly in the first period and indirectly through the social security benefit in the second period. In particular, $\partial s/\partial w = (1 - \gamma)S_1 + \beta S_2 = (1 - \gamma + \beta R)S_1 - \beta R$. It follows that if $S_1 > \beta R(1 - \gamma + \beta R)^{-1}$, then saving is increasing in the wage rate.

For the representative spender, $U^*_1/U^*_2 = u^*_1(c^*_1)/bu^*_1(c^*_2) > 1+r$ evaluated at $c^*_{1t} = I_{1t}$ and $c^*_{2t+1} = I_{2t+1}$, i.e., the indifference curve in c_1-c_2 space is steeper than the budget line at $c^*_{1t} = I_{1t}$ and $c^*_{2t+1} = I_{2t+1}$ where savings is zero.[10] In fact, the spender would like to borrow against future social security transfers under this assumption but is prohibited from doing so. It is convenient to define the inverse of the marginal rate of substitution between second and first period consumption for the spender as $R^* = U^*_2/U^*_1$, evaluated at zero savings. It follows that $R = 1/(1+r) > R^*$. Also notice that R^* can be adjusted by altering the social security parameters. For example, an increase in the benefit increases second period consumption. This reduces the marginal utility of second period consumption U^*_2 and in turn lowers R^*.

The indirect utility function of the young saver at time t is a function of I_{1t}, I_{2t+1}, r_{t+1}, and the public good at t and t+1, and is defined by,

$$V^t = u^t(w_t(1 - \gamma_t) - S(\,)) + bu^t((1+r_{t+1})S(\,) + \alpha_{t+1} + \beta_{t+1}w_t) + v^t(g_t) + bv^t(g_{t+1}).$$

It has the usual derivative properties, e.g., $\partial V^t/\partial I_{1t} = u^t_1$. In addition, $\partial V^t/\partial g_t = v^t_1(g_t)$ and $\partial V^t/\partial g_{t+1} = bv^t_1(g_{t+1})$. The marginal willingness-to-pay for the public good at t and t+1 for the agent born at t is given by $m_1^t = v_1^t(g_t)/u_1^t(c_{1t})$ and $m_2^t = v_1^t(g_{t+1})/u_1^t(c_{2t+1})$. Let V^* denote the spender's indirect utility function. It is defined in the same way as the saver ex post but with $s^* = 0$, and has similar properties. In a steady state, the marginal willingness-to-pay is m_1, m_2, m^*_1 and m^*_2 for the saver and the spender, respectively.

3.5 The initial old generation

There are N_0 old agents alive in the first period in each country. A saver in the initial old generation is endowed with $s_0 = K_1/\phi N_0$ units of capital. The capital is allocated across countries after observing all of the policy rules for that period. Both savers and spenders in the initial old generation receive a social security transfer of $\alpha_1 + \beta_1 w_0$. The indirect utility function for the initial old saver is $V^0 = u^0((1+ r_1)s_0 + \alpha_1 + \beta_1 w_0) + v^0(g_1)$, and for the initial old spender is $V^{*0} = u^{*0}(\alpha_1 + \beta_1 w_0) + v^{*0}(g_1)$. Finally, w_0 and s_0 are both fixed at time t = 1.

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9 It is straightforward to show that if the cross partial derivative U_{12} is non-negative, consumption is a normal good. It follows immediately that savings is increasing in first period income and decreasing in second period income. This certainly follows for the additively separable case.

10 The income variables I_1 and I_2 are defined in the same way for the two types.

3.6 Capital market equilibrium

The initial supply of capital is fixed by the endowment of the initial old generation. This gives us the following condition that determines r_1,

$$\phi\Sigma_j s_{j0} = (1+n)\Sigma_j K_j(r_1 + \tau_{j1}), \quad (3)$$

where the sum is across countries indexed by j. Equilibrium in the capital market for t ≥ 1 must satisfy

$$\phi\Sigma_j S_j((1 - \gamma_{jt})W_j(r_{njt}), \alpha_{jt+1} + \beta_{jt+1}W_j(r_{njt}), r_{t+1}) = (1+n)\Sigma_j K_j(r_{njt+1}). \quad (4)$$

In a symmetric, steady state equilibrium we have the following capital market clearing condition,

$$\phi S((1 - \gamma)W(r + \tau), \alpha + \beta W(r + \tau), r) = (1+n)K(r + \tau). \quad (5)$$

3.7 Government agencies

At time t = 0, the social welfare function used by both government agencies to choose policy in the representative country is given by[11]

$$SWF = [\phi V^{t-1} + (1 - \phi)V^{*t-1}]/(1 + n) + \Sigma_t[b(1 + n)]^{t-1}\{\phi V^t + (1 - \phi)V^{*t}\}, \quad (6)$$

where the sum is from t = 1 to infinity. Both government agencies use the same social welfare function as the objective function in its optimal control problem. The general government budget is

$$\tau_t K(r_t + \tau_t) = g_t, \text{ for } t = 1, 2, ... \quad (7)$$

and the social security program's budget is

$$(1+n)\gamma_t w_t = \alpha_t + \beta_t w_{t-1}, \text{ for } t = 1 , 2, ... \quad (8)$$

The GTA chooses the infinite policy sequence $\{\tau_t, g_t\}_t$ to maximize (6) subject to (7), taking as given the policy rule chosen by the SSA, the policies in all other countries, and aggregate variables like the world interest rate. The SSA chooses the infinite policy sequence $\{\gamma_t, \beta_t\}_t$ to maximize (6) subject to (8), taking as given the policy rule chosen by the GTA, the policies in all other countries, and aggregate variables like the world interest rate. The result is a set of policy rules that necessarily must be satisfied in equilibrium. Agencies in all countries choose their policy rules simultaneously under the same conditions and the rules are public knowledge.

3.9 Nash policy equilibrium

A *Nash policy equilibrium* (NPE) is an infinite sequence of interest rates $\{r_t\}_t$, general tax and spending policies, $\{\tau_{jt}, g_{jt}\}_{j,t}$, and social security policies, $\{\beta_{jt}, \gamma_{jt}\}_{j,t}$, such that

i. The savings function $S_j((1 - \gamma_{jt})W_{jt}, \alpha_{jt+1} + \beta_{jt+1}W_{jt}, r_{t+1})$ solves the representative saver's decision problem for j = 1, ..., J, t = 1, 2, ...;

ii. The capital demand function $K_j(r_t + \tau_{jt})$ solves the decision problem of the representative firm and $w_{jt} = W_j(r_t + \tau_{jt})$ for j = 1, ..., J, t = 1, 2, ...;

iii. The policy sequence $\{\tau_{jt}, g_{jt}\}_{j,t}$ solves the GTA's decision problem, the sequence $\{\beta_{jt}, \gamma_{jt}\}_{j,t}$ solves the SSA's decision problem, and equations (7) and (8) hold, for j = 1, ..., J, t = 1, 2, ...;

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[11] Following the literature, we use the ex post utility function of the spender in the social welfare function, rather than the ex ante function. In a strong sense, the ex ante function causes the saving problem for spenders to begin with. Use of the ex post function allows the government to appropriately address the policy problem and calculate the correct policy response. See the discussion in Cremer, et. al. (2008).

iv. The capital market clears so (3) holds for the initial generation for j = 1, 2, ..., J, and (4) holds for j = 1, 2, ..., J, t = 1, 2, ...,

We can define a symmetric Nash policy equilibrium in the following way. A symmetric, steady state, Nash policy equilibrium is an interest rate r, a policy for the representative GTA, (τ, g), and a policy for the representative SSA, (γ, β), such that

i. consumers behave optimally as described above;
ii. firms behave optimally as described above;
iii. the agencies choose their policies optimally as described above; and
iv. equation (5) holds.

We will make the standard assumptions that such an equilibrium exists and is unique and stable.

4 Optimal policy rules

It is well known that governments choosing policy in a decentralized setting may be unable to achieve the first-best allocation. A mechanism that ties the interest rate to the growth rate of the economy may not be available. The economy need not achieve the golden rule expansion path when decisions are decentralized. Governments may have to rely on distorting taxes to finance spending. And, there may be tension across countries when they compete for the same mobile tax base. The following proposition governs the optimal policy rules in the second-best. The rules in (9) constitute part iii) of the definition of the NPE.

Proposition 1: *The following two formulas govern the optimal policy rules in a locally stable, symmetric, steady state NPE,*

$$\Phi(m_1 + Rm_2) + (1 - \Phi)(m^*_1 + R^*m^*_2) = (1 - \theta\varepsilon)^{-1}[1 - \gamma + \beta R + \beta(1-\Phi)(R^* - R)], \quad (9a)$$

$$\phi u_1 + (1-\phi)u^*_1 = (\phi u_2 + (1-\phi)u^*_2), \quad (9b)$$

where $\Phi = \phi u_1[\phi U_1 + (1-\phi)U^*_1]^{-1} \leq 1$, $1 - \Phi = (1-\phi)u^*_1[\phi U_1 + (1-\phi)U^*_1]^{-1} < 1$, $\theta = \tau/r_n < 1$, *and* $\varepsilon = -(r_n/k)K_r > 0$ *is the user cost of capital elasticity.*
Proof: See Appendix A.

Equation (9a) is the modified Samuelson rule for the local public good and equation (9b) captures the optimal choice of the social security parameters. The left hand side of the modified Samuelson rule captures the present value of the sum of the marginal benefits of the public good, while the right hand side is the social marginal cost of the public good. The term $(1 - \theta\varepsilon)^{-1} > 1$ can be interpreted as the marginal cost of public funds in the absence of social security.[12] Equation (9b) states that the social security parameters should be chosen to equalize the marginal utilities of first and second period consumption of the private good when averaged across types.

There are two new elements in the modified Samuelson rule due to the presence of the social security program. First, the capital tax will alter the wage and

[12] Estimates of the user cost elasticity by Caballero, et. al. (1995) using US data range from 0.01 to 2.0 depending on the industry. Hassett and Hubbard (2002) give a range of 0.5 to 1.0. Schaller (2006) estimates the long run user cost elasticity for equipment to be well above one in magnitude and essentially zero for structures using Canadian time series data and cointegration techniques. If $\theta\varepsilon < 1$, $(1 - \theta\varepsilon)^{-1} > 1$.

this will affect welfare through the social security parameters according to $\beta/(1+r) - \gamma$. Using the social security program's budget this term becomes,

$$\beta/(1+r) - \gamma = - (r - n)\beta R/(1+n) - \alpha/w(1+n).$$

This expression is negative if $r \geq n$ and $\alpha \geq 0$, and serves to raise the social marginal cost of providing the public good. On the other hand, if $\alpha < 0$ and $n > r$, this will tend to lower the social cost of the public good.[13] Second, the imperfect policy tools available may affect the distribution of welfare across types. While both types may indirectly bear some of the burden of the capital tax through lower wages, savers may bear an additional burden through the interest rate and this may affect the distribution of welfare favorably for spenders. This effect is captured by the term, $\beta(1 - \Phi)(R^* - R) < 0$ in (9a), which tends to reduce the social cost of the public good. Intuitively, the saver achieves a higher level of welfare since they recognize the need to smooth their intertemporal consumption. The extra burden they bear due to the capital tax alters the distribution of welfare favorably for spenders.

5 Response to a coordinated tax reform experiment

In this section we will study the positive implications of a general increase in the capital tax rate and the normative implications in the next section. We imagine the economy is in a symmetric steady state Nash policy equilibrium where $g, \tau, \beta, \gamma > 0$ in each country and spenders do not save privately, $s^* = 0$. The countries collectively agree to a permanent increase in the capital tax rate, and this is publicly announced and implemented at a given point in time, say time $t = 1$. All agents believe the announcement and change their expectations accordingly. The purpose of the reform is to alleviate the policy externality due to horizontal tax competition across countries.

We assume the payroll tax rate adjusts in a manner that is uncoordinated across countries so the social security program's budget balances under this policy reform exercise, while the other social security parameters are fixed. Proposition 2 would not be substantially altered if one of the other social security parameters responded instead.[14]

Proposition 2: *Consider a capital tax reform such that $d\tau_{jt} = d\tau > 0$ for all j and all t ≥ 1. The response to the coordinated increase in the capital tax rate in every country is the following.*

(i) The capital tax reform is completely capitalized into the interest rate in the first period, $dr_1/d\tau = -1$, and the local wage and payroll tax rate are unaffected in the first period.

(ii) The interest rate and the local wage fall in the second period and the tax reform is not completely capitalized. The payroll tax rate increases in the second period to balance the social security budget.

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[13] One issue that has arisen in the public goods literature is whether the second-best social cost of the public good is greater than or less than one in magnitude since this may have implications for the level of the public good relative to the first-best case. It is possible for the right hand side of (9a) to be less than one in magnitude in our model if $(\gamma - \beta/1+r) < \theta\varepsilon$, since $R^* < R$. It is also possible that the right hand side is equal to one, as well. I am indebted to an anonymous referee for this observation.

[14] The analysis in this section is positive. In the normative welfare analysis of the reform in the next section, terms in $d\gamma_t/d\tau$ drop out of the formula when use is made of the optimal policy rules in Proposition 1 and the envelope theorem is applied.

(iii) The interest rate and the local wage fall in the steady state and the tax reform is not completely capitalized in the long run. The payroll tax rate rises in the steady state when $(1+n)\gamma > \beta$.

(iv) The interest rate and the local wage fall on the transition path. The magnitude of the response of the interest rate (local wage) diminishes (increases) as it converges to the steady state response. The tax reform is not completely capitalized on the transition path. The payroll tax rate rises on the transition path and the magnitude of the response decreases as it converges to the steady state response.

Proof: See Appendix B.

5.1 Initial response

Equation (3) and equation (8) for t = 1 form a recursive two equation system in r_1 and γ_1. It is immediate from equation (3) that the capital tax increase is completely capitalized in the interest rate, $dr_1/d\tau = -1$. This follows since the supply of capital at t = 1 is fixed by past savings decisions and the same tax increase occurs in all countries. The local wage in the first period is unaffected as a result of the complete capitalization of the reform since the direct effect of the reform that works through the tax rate cancels the indirect effect that works through the interest rate when the supply of capital is fixed. To see this note that $w_1 = W(r_1+\tau_1)$ so the response to the reform is $dw_1/d\tau = -k(1+dr_1/d\tau_1) = 0$. The first term - k is the direct effect while the second term in $dr_1/d\tau_1$ is the indirect effect. Since the local wage in the first period is unaffected by this reform, it is immediate that the social security program's budget in each country will not be affected in the first period so that $d\gamma_1/d\tau = 0$.

5.2 Response in the second period

Next, consider the initial young generation at t = 1. They confront equations (4) for t = 1 and (8) for t = 2, a system which is not recursive. The interest rate and the wage fall and the payroll tax rate rises in the second period in response to the reform. The reform is not completely capitalized. The intuition is that as the capital tax rate rises in the second period, the cost of capital increases making capital less desirable as an input relative to labor. Demand for capital falls and the interest rate falls as a result. Capitalization is incomplete since the supply of capital is endogenous in the second period. The wage also falls in the second period and as the wage falls, payroll tax revenues $\gamma_2 W(r_2 + \tau_2)$ fall because the wage base is falling. The tied benefit $\beta_2 W(r_1 + \tau_1)/(1+n)$ remains constant, however, since the wage in the first period is unaffected by the reform. The payroll tax rate γ_2 must rise in the second period to maintain the program's budget. The response of γ_2 can be interpreted as an endogenous reform of the social security program required to maintain the cash flow into the program.

5.3 Steady state response

The tax reform reduces the demand for capital per worker in every country in the long run. The wage in each country and the interest rate fall in response. However, the impact of the reform on the social security program depends on whether there is a universal benefit, or not. When the wage falls, payroll tax revenues fall by $(1+n)\gamma(dw/d\tau)$, while tied benefits fall by $\beta(dw/d\tau)$. If there is a universal benefit, it must be financed through the payroll tax, hence $\gamma > \beta/(1+n)$ from equation (8). This means that the decline in payroll tax revenues will outweigh the decline in the tied benefit in the long run, which is why the payroll tax rate rises in the long run.

5.4 Transition

The capital tax is completely capitalized in the interest rate in the first period and the wage and the payroll tax rate do not respond. The interest rate in the second period falls by less than in the first period, the wage in the second period begins to fall, and the payroll tax rate in the second period begins to rise. These responses continue on the transition path as the economy converges to the steady state response. It is shown in Appendix B that the response of the interest rate follows a second order difference equation that can be solved to obtain[15]

$$dr_t/d\tau = dr/d\tau - (1 + dr/d\tau)(1 - \Delta/D)(\lambda_1)^{t-2}, \qquad (11)$$

for t = 2, 3,....., where $dr/d\tau$ is the steady state response, λ_1 is the stable root of the dynamical system, $0 < \lambda_1 < 1$, $D = \phi S_r - (1+n)K_r > 0$, $A_1 = k\phi(S_1 + \beta S_2)/D$, and $\Delta = D(1 - A_1)$. If $\Delta > 0$, the equilibrium is stable. We provide a simple condition in the Appendix such that $A_1 < 1$, and hence $\Delta > 0$. In addition, we show that $D > \Delta$. As t increases, the term in λ_1 diminishes in magnitude and the sequence converges to the steady state response. As a result, the sequence $\{dr_t/d\tau\}_t$ is negative, decreases in magnitude, and converges to the steady state response. The local wage falls on the transition path and the response increases in magnitude as it converges to the steady state response,

$$0 = (dw_1/d\tau) > (dw_2/d\tau) > ... > (dw_t/d\tau) > ... > (dw/d\tau).$$

Finally, the payroll tax rate increases on the transition path as the wage falls but the magnitude of the increase diminishes,

$$(d\gamma_2/d\tau) > ... > (d\gamma_t/d\tau) > ... > (d\gamma/d\tau) > 0 = d\gamma_1/d\tau.$$

The magnitude diminishes because the need for raising the payroll tax rate is reduced in the transition since total benefits fall as the wage falls.

We can also consider the case where the universal benefit is absent. When this is true the social security program's budget becomes, $\gamma_t w_t = \beta_t w_{t-1}/(1+n)$ and in a steady state $\gamma = \beta/(1+n)$. This gives us the following corollary to the proposition.

Corollary 1: *Consider a capital tax reform such that $d\tau_{jt} = d\tau > 0$ for all j and all t, where the universal benefit is absent, $\alpha_{jt} = 0$ for all t and all j. The response to the coordinated increase in the capital tax rate at every location is the same in the first period, the second period, and on the transition path, as in Proposition 2. However, the payroll tax rate does not respond in the steady state when $\gamma = \beta/(1+n)$.*

6 Welfare effects of the coordinated capital tax reform

The structure of the overlapping generations model delivers a broad variety of new welfare effects that extend those obtained in static or two period models. Not all agents will experience each effect. In particular, the effects confronted by the initial

[15] See Bernheim (1981) for this method of writing the response as a combination of short and long run adjustments. The first term captures the long run adjustment while the second term in λ_1 captures the short run adjustment. As time proceeds the emphasis shifts from the short run to the long run term.

old generation differ from subsequent generations. We should also note that the only welfare effect that arises in the static capital allocation model under symmetry, or its two period extension, is the current spending effect.[16]

Proposition 3: *Suppose the capital tax rate increases in all countries for all periods, $d\tau_{jt} = d\tau > 0$ for $j = 1, ..., J$; $t = 1, 2, ...$ If the following conditions hold*

(i) $\theta \varepsilon m_2^t > s/k = (1+n)/\phi$ *for savers for t = 1, 2,..., ∞;*

(ii) $-(1+\theta \varepsilon m_1^t)(dr_t/d\tau) > \gamma_t - \beta_{t+1}/(1+r)$ *for savers for t = 1, 2, ..., ∞;*

(iii) $-(1+\theta \varepsilon m_1^{*t})(dr_t/d\tau) > \gamma_t - \beta_{t+1}/(1+r)$ *for spenders for t = 1, 2, ..., ∞;*

then everyone in the initial generation, generations on the transition path, and those living in the steady state, are better off as a result of the coordinated reform, where the welfare effects are evaluated in the initial symmetric steady state NPE.
Proof: See Appendix C.

The reform causes direct effects and indirect effects that work through the wage and interest rate. The direct effects drop out of the welfare formulas when use of the optimal policy rules of Proposition 1 is made and the envelope theorem is applied. Indirectly, the reform will generally cause a spending effect in both periods of life, a wage effect when young, a wage effect that works through social security, and an interest income effect for savers. The wage effect when young occurs because the fall in the interest rate lowers the user cost of capital. This indirectly shifts the demand by firms toward capital and raises the wage indirectly. The wage effect working through social security depends on whether there is a universal benefit, or not. If there is, then payroll taxes paid into the system increase more than total benefits when the wage rises thus reducing welfare. The opposite is true if there is a lump sum element included as part of the tax that supports the program. Condition (i) of Proposition 3 compares the positive spending effect to the negative interest income effect for the savers when old. Conditions (ii) and (iii) compare the positive spending and first period wage effects with the negative wage effect working through social security.

6.1 Initial old generation

The initial old generation of spenders is unambiguously better off since they only experience the positive spending effect. The initial old generation of savers experiences the positive spending effect but a negative interest income effect; interest income falls as the interest rate falls in response to the reform. They are better off if the former effect dominates the latter, which occurs when condition (i) of Proposition 3 holds for t = 1. This condition is more likely to hold the larger m_2 is, the larger $\phi\theta\varepsilon$ is, and the smaller the population growth rate is.

6.2 Initial young generation

The welfare effects of the reform for the initial young generation include a positive spending effect when young, a positive spending effect when old, a positive wage effect, a wage effect on the social security program that takes the sign of - (γ_1 - $\beta_2/(1+r)$), and for the savers a negative interest income effect when they are old. Welfare for the spender improves if condition (iii) holds for t = 1 since the spending

[16] This is demonstrated in the Appendix in Batina (2009).

effect outweighs the effect working through social security under that condition. The young saver is better off under the reform if conditions (i) and (ii) hold for t = 1. In that case, the current spending effect outweighs the social security wage effect and the future spending effect outweighs the interest income effect. If there is a universal benefit, the payroll taxes will be greater than the tied benefits. In that case, the wage effect working through social security reduces the welfare gains from the reform.

6.3 Steady state impact

Both types of agent experience the spending effect when young and the future spending effect when old, which both cause an improvement in welfare. Both savers and spenders also experience the wage effect, which serves to improve welfare, and the social security wage effect, which is negative if there is a universal benefit. Finally, savers experience the negative interest income effect. Under the conditions of the proposition welfare improves for both types. Spenders benefit more at the margin since they do not experience the negative interest income effect. The social security wage effect reduces the gains to the reform for both type of agent when there is a universal benefit.

6.4 Transition

The welfare effects of the reform on the transition path track those of the first young generation and subsequent generations living in the steady state. All of the effects are positive for the spenders except the social security wage effect. All of the effects are positive for savers except the interest income effect and the wage effect working through social security. If the conditions of the proposition hold on the transition path, then both types of agent will be better off because of the reform.

It is interesting to note that the magnitude of the interest income effect will fall on the transition path as the economy adjusts since the magnitude of the response of the interest rate falls on the transition path. The same will be true for the indirect wage effects since that effect is given by - $k(dr_t/d\tau)$, which is positive but decreasing in magnitude as well.

6.5 Discussion

How does social security affect the welfare results? If social security depends on wages and the payroll tax revenue is greater than the tied benefit, the wage effect working through social security is negative in sign. This makes it less likely that the capital tax reform will improve welfare. Second, if social security does not depend on wages, the right hand side of conditions (ii) and (iii) are zero and the conditions hold automatically. Thus, social security makes it harder to satisfy the conditions of the Proposition 3. Of course, satisfying the sufficient conditions doesn't tell us about the magnitude of the impact of social security on the reform. In the next section we present a simulation example where we calculate the impact of the reform on the steady state equilibrium and the welfare effects of the reform.

7 Example

We simulate the long run equilibrium of the model under symmetry and consider a simple reform where there is a coordinated 20% increase in the capital tax rate across all countries. We will assume the social security payroll tax and benefit rate adjust optimally to the reform so equations (8) and (9b) hold. The response of the social security parameters is undertaken in a manner that is uncoordinated across countries.

Suppose production is Cobb-Douglas, $y = Ak^a$ and utility for both types is $U = Ln(c_{1t}) + \eta Ln(g_t) + b[Ln(c_{2t+1}) + \eta Ln(g_{t+1})]$. The parameters are (η, ϕ, a, A, b, n). We chose the parameters to generate a wage income share of 2/3 of GDP, a real annual interest rate of 3.5%, a ratio of the universal benefit to total benefits of 0.20 $(\alpha/(\alpha + \beta w) = 0.20)$, a public good spending to GDP ratio of 10% (g/gdp = 0.1), and an annual population growth rate of 1.27%, the average rate since 1900 for the US. The pattern of results was not affected by small changes in the parameters, or by choosing the parameters to generate a government spending to GDP ratio of 5% or 15%. In each case the second-best level of government spending is less than the first-best level, before and after the reform. We also simulate the model using the same parameters except that n = 0.[17]

The results are listed in Tables 1 and 2. The top part of Table 1 depicts the positive effect of the reform, while the bottom part gives the impact of the reform on welfare in the short run for the initial old and young agents alive when the reform is first introduced and for the agents living in the new long run equilibrium. The welfare effects for the new generations on the transition path are similar to those living in the new steady state. In the reform we assume there is a 20% increase in the tax rate in all countries and allow the other variables, (s, k, g, γ, β) to be chosen optimally given the higher capital tax rate. We should emphasize that the social security parameters (γ, β) respond optimally in an uncoordinated manner across countries under the reform of the capital tax. The gain in welfare is the percent change in utility in response to the reform.

Table 1: Simulation results

	annual r	wage	capital tax rate	payroll tax rate	benefit rate	
baseline	0.035	10.52	0.3	0.2	0.233	
20% increase	0.0326	10.43	0.36	0.205	0.239	
Welfare effects of the reform	initial old spender	initial old saver	initial young spender	initial young saver	spender	saver
% gain in welfare	0.0423	-0.0017	0.0810	0.0152	0.0222	0.0075

The reform causes the user cost of capital to rise in all countries and reduces the demand for capital relative to labor. Both the interest rate and the wage fall in the long run in response to the reform. Second, both the payroll tax rate and the benefit rate of the social security program rise in response to the reform. This compensates both types of agent for the decrease in the wage. The increase in the social security

[17] The parameters are b = 0.5275, $\eta = 0.2008$, a = 0.334, A = 14.5, $\phi = .65$, $\alpha = .6175$. The steady state equilibrium before the reform is g/gdp = 0.1, $\tau/(r+\tau) = 0.3$, annual r = 0.035, $(\alpha+\beta w)/w$ = replacement income = 0.292, and $\alpha/(\alpha+\beta w) = 0.2$. After the reform, g/gdp = 0.12, $\tau/(r+\tau) = 0.36$, annual r = 0.0326, $(\alpha+\beta w)/w = 0.298$, and $\alpha/(\alpha+\beta w) = 0.1985$.

tax and benefit also shifts income to the retirement period of the life cycle to offset the fall in saving. As this occurs the replacement ratio $(\alpha + \beta w)/w$ increases from 29.2% to 29.8%. Ironically, raising the capital tax rate to alleviate the externality associated with the tax on mobile capital may induce an increase in the payroll tax rate and the social security benefit which in turn reduces savings causing spillover effects for other countries that may reduce the supply of world capital and lower wages in other countries as a result.

In terms of the welfare effects, the initial old spender and initial young spender are both better off since they only experience the positive effect associated with the increase in public spending. Since the initial young spender experiences this effect when young and old, they experience a greater improvement in welfare than the initial old spender. The initial young spender actually benefits the most from the reform. The initial old saver experiences the positive effect of the increased public spending but the negative interest income effect, and is actually worse off since the latter effect dominates the former. Under the simulation exercise, the sufficient condition (i) of Proposition 3 for t = 1 is violated $(\theta\varepsilon m_2 - s/k = - 1.246 < 0)$ signaling that the initial old savers may be worse off as a result of the reform, and this turns out to be the case. The initial young saver experiences the spending effect when young and when old and the interest income effect, but is better off since the combined spending effects outweigh the interest income effect. Condition (ii) holds in this case $(- (1 + \theta\varepsilon m_1)(dr_1/d\tau) - (\gamma - \beta/(1+r)) = 1.27 > 0)$. Finally, the spender and saver living in the new steady state are both better off. However, the increase in utility is smaller for them than the initial young agents of their respective type because they also experience the negative social security wage effect, which reduces the welfare gain to the reform.

Table 2: Sensitivity Analysis

% change in welfare of the reform	initial old spender	initial old saver	initial young spender	initial young saver	spender	saver
ϕ = .65 (Table 1)	0.0423	-0.0017	0.0810	0.0152	0.0222	0.0075
ϕ = .5	0.0464	0.0014	0.102	0.0152	0.0312	0.0106
g/gdp = 0.15	0.0783	0.0074	0.1285	0.0400	0.0302	0.0142
n = 0	0.0325	-0.0044	0.0589	0.0141	0.0107	0.0012

In Table 2 we depict the welfare effects associated with the reform under different assumptions on some of the key parameters, the percent of life cycle savers, the level of government spending on the public good relative to GDP, and the population growth rate. In the first row of the table we repeat the earlier result from Table 1 for convenience. The initial equilibrium is the same across all of the experiments, r = 0.035, g/gdp = 0.1, $\tau/(r + \tau) = 0.3$, and $\alpha/(\alpha + \beta w) = 0.2$, and a 20% increase in the tax rate is imposed.

The same patterns emerge in the response of the endogenous variables to the reform as in Table 1, however, the welfare effects change in magnitude significantly for most individuals. First, when there are fewer life cycle savers (row 2), everyone is better off under the reform including the initial saver and the magnitude of the welfare improvement increases across all consumers since the wage does not fall as much as when there are more life cycle agents. One can see this from the welfare impact of the initial young saver whose wage does not change under the reform. And savings income does not fall as much for the initial savers so they are better off as well. Second, all agents benefit from the reform and the magnitude of the improvement in welfare is also greater when the initial spending to GDP ratio is higher (row 3). Social security has less of a detrimental impact when it is smaller relative to general government spending. Finally, the improvement in welfare is smaller for each agent when the population is not growing (row 4). This is telling since many countries in the OECD are experiencing slowing population growth. The welfare gains in such economies may not be as large as in countries where the population is growing more rapidly.

8 Conclusion

We extend the literature on capital tax competition to a dynamic overlapping generations economy where separate government agencies choose general tax policy and social security policy, where a source based capital tax is used to finance a public good, and where a forced saving social security program is imposed on a pay-as-you-go basis that depends on wages. Our focus is on studying the impact of a coordinated capital tax reform where all countries raise their capital tax rate and spend the proceeds in an attempt to overcome the policy externality problem. A number of new welfare effects are uncovered, and we derive sufficient conditions such that all generations are better off with the reform. The impact of such a capital tax reform on welfare will depend on the parameters of the social security program and myopic spenders may enjoy a greater benefit from such a reform than life cycle savers, both when the reform is first imposed and also in the long run.

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Appendix

A. Proposition 1: Optimal second-best policy rules

Writing out the social welfare function of equation (6),

$$\{\phi[u^0((1+r_1)s_0 + \alpha_1 + \beta_1 w_0) + v(g_1)] + (1-\phi)[u^{*0}(\alpha_1 + \beta_1 w_0) + v^*(g_1)]\}/(1+n)$$
$$+ \Sigma[b(1+n)]^{t-1}\{\phi[u^t(w_t(1-\gamma_t) - s_t) + v^t(g_t) + b(u^t((1+r_{t+1})s_t + \alpha_{t+1} + \beta_{t+1}w_t) + v^t(g_{t+1}))]$$
$$+ (1-\phi)[u^{*t}(w_t(1-\gamma_t)) + v^*(g_t) + b(u^{*t}(\alpha_{t+1} + \beta_{t+1}w_t) + v^{*t}(g_{t+1}))]\}.$$

First, consider the optimal policy rule of the GTA. It chooses an infinite sequence $\{\tau_t, g_t\}_t$ to maximize the social welfare function subject to a sequence of budget constraints given by equation (7), taking as given the policy sequence of the other agency, the policy sequences of all other countries, and the sequence of interest rates. The first order conditions for the optimal policy sequence at time t are given by,

$$[b(1+n)]^{t-1}\{b(\phi v^{t-1}{}_1(g_t) + (1-\phi)v^{*t-1}{}_1(g_t)) + (1+n)b[\phi v^t{}_1(g_t) + (1-\phi)v^{*t}{}_1(g_t)]\} - \lambda_t = 0,$$
$$\lambda_t(k + \tau K_r) - k[b(1+n)]^{t-1}\{(1-\gamma)(\phi u^t{}_1(c_{1t}) + (1-\phi)u^{*t}{}_1(c^*{}_{1t}))$$

$$+ b\beta[\phi u^t{}_2(c_{2t+1}) + (1-\phi)u^{*t}{}_2(c^*{}_{2t+1})]\} = 0,$$

where λ_t is the Lagrange multiplier for its budget constraint at time t. Combine conditions to obtain

$$\phi v^{t-1}{}_1(g_t) + (1-\phi)v^{*t-1}{}_1(g_t) + (1+n)[\phi v^t{}_1(g_t) + (1-\phi)v^{*t}{}_1(g_t)] =$$
$$(1 - \theta\varepsilon)^{-1}\{\phi[u^t{}_1(1-\gamma) + b\beta u^t{}_2(c_{2t+1})] + (1-\phi)[u^{*t}{}_1(c^*{}_{1t})(1-\gamma) + b\beta u^{*t}{}_2(c^*{}_{2t+1})]\}, \quad \text{(A1)}$$

Use $u_1 = b(1+r)u_2$, $u^*{}_1 = b(1+r^*)u^*{}_2$, the definition of Φ, and evaluate at the initial steady state, to rewrite (A1) as equation (9a).

The first order conditions for an interior solution for the SSA's decision problem are,

$$[b(1 + n)]^{t-1}[\phi u^t{}_1(c_{1t}) + (1-\phi)u^{*t}{}_1(c^*{}_{1t})] = \mu_t(1+n),$$
$$[b(1 + n)]^{t-2}[\phi u^{t-1}{}_1(c_{2t}) + (1-\phi)\, u^{*t-1}{}_1(c^*{}_{2t})]b = \mu_t,$$

where μ is the Lagrange multiplier for the social security constraint. Combine these conditions and evaluate at the initial steady state to obtain (9b).

B. Proposition 2: Response to the capital tax policy reform

B.1 Preliminary results

Solve the social security constraint for the tax rate, $\gamma_t = \alpha/w_{t-1}(1+n) + \beta_t/(1+n)$, substitute into the savings function in the steady state equilibrium condition, and differentiate with respect to r to obtain $\Delta = \phi S_r - (1+n)K_r - \beta\phi k S_2 - [1 - \beta/(1+n)]k\phi S_1$. This can also be written as $\Delta = D + k\phi[\beta R - [1 + \beta R - \beta/(1+n)]S_1]$, by using $S_2 = -R(1-S_1)$, where $D = \phi S_r - (1+n)K_r > 0$. If $\Delta > 0$, then the steady state equilibrium is stable when it exists. Savings is increasing in the wage if and only if

$$S_1 > \beta R(1 + \beta R - \gamma)^{-1}. \quad \text{(B1)}$$

Furthermore, when $\alpha > 0$, it follows that $\gamma > \beta/(1+n)$, from the social security budget constraint. Thus, $1 + \beta R - \gamma < 1 + \beta R - \beta/(1+n)$, or,

$$[1 + \beta R - \gamma]^{-1} > [1 + \beta R - \beta/(1+n)]^{-1}. \quad \text{(B2)}$$

If (B1) is satisfied, then $S_1 > \beta R[1 + \beta R - \beta/(1+n)]^{-1}$, from (B2). It then follows that $\phi[\beta R - [1 + \beta R - \beta/(1+n)]S_1] < 0$. Hence $D > \Delta$.

B.2 Initial response

Equations (3) and (8) for t = 1 determine (r_1, γ_1). Equation (3) implies that $dr_1/d\tau = -1$ and w_1 is unaffected by the reform. It follows from this and (8) that $d\gamma_1/d\tau = 0$. To see this consider the social security budget in the first period, $(1+n)\gamma_1 W(r_1 + \tau_1) = \alpha_1 + \beta_1 w_0$. The right hand side is fixed and the $d(r_1 + \tau_1)/d\tau_1 = 0$. It follows immediately that $d\gamma_1/d\tau = 0$. The young at t =1 face equation (4) for t = 1 and equation (8) for t = 2. Differentiate this system and solve to obtain the following,

$$dr_2/d\tau = -1 + \phi S_r/D < 0, \quad \text{(B3)}$$
$$d\gamma_2/d\tau = (k\gamma/w)[1 + (dr_2/d\tau)] > 0, \quad \text{(B4)}$$

where we have used $dr_1/d\tau = -1$. Note that $0 < \phi S_r/D < 1$ since $K_r < 0$. The wage rate in the second period, $W(r_2 + \delta + \tau_2)$, responds according to

$$dw_2/d\tau = - k(1+ dr_2/d\tau) = - \phi k S_r/D < 0,$$

where we have used $W_r = - k$.

B.3 Steady state response

Substitute the steady state social security budget for the payroll tax rate into the equilibrium condition to obtain,

$$\phi S(W(r + \tau)[1 - \beta/(1+n)] - \alpha/(1+n), \alpha + \beta W(r + \tau), r) = (1+n)K(r + \tau).$$

To derive the steady state responses, totally differentiate this last equation to obtain,

$$dr/d\tau = -1 + \phi S_r/\Delta < 0. \qquad \text{(B5)}$$

Since $\Delta < D$, it follows that $dr_2/d\tau < dr/d\tau < 0$. The response of the payroll tax rate is,

$$d\gamma/d\tau = (k/w)[\gamma - \beta/(1+n)][1 + (dr/d\tau)] > 0 \qquad \text{(B6)}$$

Under (B1), $d\gamma/d\tau < d\gamma_2/d\tau$. To see this compare (B4) with (B6) after using (B3) and (B5). The local wage $W(r + \delta + \tau)$ responds to the reform according to

$$dw/d\tau = - k(1 + dr/d\tau)) = - \phi k S_r/\Delta < 0.$$

Since $D > \Delta$, $dw/d\tau < dw_2/d\tau$.

B.4 Transition response

Substitute the social security constraint into the equilibrium condition for the payroll tax to get,

$$\phi S(W(r_t + \tau) - (\alpha + \beta W(r_{t-1} + \tau))/(1+n), \alpha + \beta W(r_t, \tau), r_{t+1}) = (1+n)K(r_{t+1} + \tau).$$

This is a second order difference equation. Differentiate,

$$D(dr_{t+1}/d\tau) - k\phi[S_1 + \beta S_2](dr_t/d\tau) + k\phi\beta(S_1/(1+n))(dr_{t-1}/d\tau) = \phi S_r - \Delta.$$

Divide by D and rewrite using the lag operator, L, where $Lx_t = x_{t-1}$,

$$(1 - A_1L + A_2L^2)(dr_{t+1}/d\tau) = A_3,$$

where $A_1 = k\phi[S_1 + \beta S_2]/D = \phi k((1 + \beta R) S_1 - \beta R)/D$, $A_2 = k\phi\beta S_1/(1+n)D$, and $A_3 = [\phi S_r - \Delta]/D$. Following Bernheim (1981), the A_i are evaluated at the initial steady state equilibrium and hence constant. If $A_1 < 1$ in the absence of social security, then the equilibrium is locally stable. If $A_1 < \beta/\gamma(1+n)$ (< or =) 1 in the presence of the social security program, then the equilibrium will be stable. The roots to this second order equation are given by

$$\lambda = (1/2)[A_1 \text{ (+ or -) } (A_1^2 - 4A_2)^{1/2}].$$

If $A_1^2 - 4A_2 > 0$, the roots are real, which is the case we will pursue. It is straightforward to show that the stable root is given by $\lambda_1 = (1/2)[A_1 + (A_1^2 - 4A_2)^{1/2}]$. If $\Delta > 0$, then $\lambda_1 < 1$. This is easy to confirm by expanding λ_1. Since $1 + \beta R - \beta/(1+n) < 1 + \beta R$, it follows that $S_1 > \beta R/(1 + \beta R)$ by (B1) so that $A_1 > 0$. Therefore, $0 < \lambda_1$. The stable solution to the difference equation is given by

$$dr_t/d\tau = dr/d\tau - (1 + dr/d\tau)(1 - \Delta/D)(\lambda_1)^{t-2} \qquad \text{(B7)}$$

or,

$$1+ dr_t/d\tau = (1+ dr/d\tau)(1 - (1 - \Delta/D)\lambda_1^{t-2}),$$

for t = 2, ..., where we have used (B3) and (B5). Clearly, the response converges to the steady state response, $dr/d\tau$. Thus,

$$- 1 = dr_1/d\tau < dr_2/d\tau < ...< dr_t/d\tau < ... < dr/d\tau < 0.$$

Since $dw_t/d\tau = - k(1 + dr_t/d\tau)$, we also have the result that

$$dw_1/d\tau = 0 > dw_2/d\tau > ... > dw_t/d\tau > .... > dw/d\tau.$$

The response of the payroll tax rate on the transition path is

$$d\gamma_t/d\tau = (k/w)[\gamma(1 + dr_t/d\tau) - (\beta/(1+n))(1 + dr_{t-1}/d\tau)],$$

from the social security budget constraint. Since $\gamma > \beta/(1+n)$ for $\alpha > 0$, and $1 + dr_t/d\tau > 1 + dr_{t-1}/d\tau$, by (B7), this response is positive, $d\gamma_t/d\tau > 0$. Furthermore, if $A_1 < \beta/(1+n)\gamma < 1$, then $d\gamma_2/d\tau > d\gamma/d\tau$. The response of the payroll tax rate converges to the steady state response and we have the result, $d\gamma_2/d\tau > ... > d\gamma_t/d\tau > ... > d\gamma/d\tau > 0$. When $\alpha = 0$, we obtain, $d\gamma_2/d\tau > ... > d\gamma_t/d\tau > .. > d\gamma/d\tau = 0$.

C. Proposition 3: Welfare effects of the coordinated capital tax reform

C.1 Initial Generations

The per capita sum of the utility of the agents alive in the first period of the reform is,

$$\Omega_1 = \phi(U^0/(1+n) + U^1) + (1-\phi)(U^{*0}/(1+n) + U^{*1}) + \mu_1[(1+n)\gamma_1 w_1 - \beta_1 w_0 - \alpha] + \mu_2[(1+n)\gamma_2 w_2 - \beta_2 w_1 - \alpha]. \quad (C1)$$

where we have appended the first two social security budget constraints since they affect utility in the first period of the economy. The Lagrange multipliers are μ_t. Substitute the GTA's budget constraint for g_1 and g_2 into (C1). Differentiate (C1) and use the definitions of θ and ε, $dr_1/d\tau = -1$, $\phi s_0 = (1+n)k_1$, and the policy rules of Proposition 1 to obtain the effect of the reform on social welfare for the initial generations,

$$\begin{aligned} d\Omega_1/d\tau = {} & (1-\phi)\theta\varepsilon U^{*0}{}_4 + \phi(k_1\theta\varepsilon m^0{}_2 - s_0)U^0{}_2 \\ & + \phi k_1[\theta\varepsilon m^1{}_1 + 1 - (\gamma_1 - \beta_2/(1+n))]U^1{}_1 \\ & + (1-\phi)k_1[\theta\varepsilon m^{*1}{}_1 + 1 - (\gamma_1 - \beta_2/(1+n))]U^{*1}{}_1 \\ & + \{\phi(s_1 - k_2\theta\varepsilon m^1{}_2)U^1{}_2 - k_1(1-\phi)U^{*1}{}_4\theta\varepsilon\}(dr_2/d\tau), \end{aligned}$$

evaluated at the initial steady state. The initial old spender is better off because of the spending effect that occurs in the first period of the reform, $\theta\varepsilon U^{*0}{}_4 > 0$. The initial old saver is better off if the positive spending effect outweighs the negative interest rate effect, which occurs if $\theta\varepsilon m^0{}_2 > s/k = (1+n)/\phi$. The initial young spender is better off if

$$1 + \theta\varepsilon m^{*1}{}_1 - \theta\varepsilon R^* m^{*1}{}_2(dr_2/d\tau) > \gamma_1 - \beta_2/(1+n),$$

where $R^* m^{*1}{}_2 = (U^{*1}{}_4/U^{*1}{}_2)(U^{*1}{}_2/U^{*1}{}_1)$. A sufficient condition is that the current spending effect outweigh the social security effect on wages, $\theta\varepsilon m^{*1}{}_1 > \gamma_1 - \beta_2/(1+n)$.

The initial young saver is better off under the reform if

$$k_1[1 + \theta\varepsilon m^1{}_1 - (\gamma_1 - \beta_2/(1+n))] + (s_1 - k_2\theta\varepsilon m^1{}_2)R(dr_2/d\tau) > 0.$$

Sufficient conditions for this to be satisfied are $\theta\varepsilon m^1{}_1 > \gamma_1 - \beta_2/(1+n)$ and $\theta\varepsilon m^1{}_2 > s/k = (1+n)/\phi$.

C.2 Welfare effects on the transition path

Define Ω_t in the same manner as Ω_1 in (C1). It is then straightforward to derive the following result, by using the same steps,

$$\begin{aligned} d\Omega_t/d\tau = {} & - k\{[\phi(U^{t-1}{}_4 + U^t{}_3) + (1-\phi)(U^{*t-1}{}_4 + U^{*t}{}_3)]\theta\varepsilon - (1+n)U^{t-1}{}_2 \\ & + [\phi U^t{}_1 + (1-\phi)U^{*t}{}_1]\}(dr_t/d\tau) + k\{(1+n)U^t{}_2 \\ & - [\phi U^t{}_4 + (1-\phi)U^{*t}{}_4]\theta\varepsilon\}(dr_{t+1}/d\tau)] - k[\phi U^t{}_1 + (1-\phi)U^{*t}{}_1](\gamma - \beta/(1+r)). \end{aligned}$$

The old spender on the transition path is unambiguously better off since they only experience the current spending effect, $-\theta\varepsilon U^{*t-1}{}_4(dr_t/d\tau) > 0$. The old saver is better off if the spending effect outweighs the interest rate effect, $\theta\varepsilon U^{t-1}{}_4 > (1+n)U^{t-1}{}_2/\phi$, or $\theta\varepsilon m^{t-1}{}_2 > (1+n)/\phi$. The welfare of the young saver improves if $(1+n)U^t{}_2 < \phi U^t{}_4\theta\varepsilon$, or

$$\theta\varepsilon m^t{}_2 > (1+n)/\phi, \quad (C2)$$

and, $-(U^t{}_3\theta\varepsilon + U^t{}_1)(dr_t/d\tau) > U^t{}_1(\gamma - \beta/(1+r))$, or

$$-(1 + m^t{}_1\theta\varepsilon)(dr_t/d\tau) > \gamma - \beta/(1+r). \quad (C3)$$

The young spender's welfare improves if $-(U^{*t}_3\theta\varepsilon + U^{*t}_1)(dr_t/d\tau) > U^{*t}_1(\gamma - \beta/(1+r))$, or

$$-(1 + m^{*t}_1\theta\varepsilon)(dr_t/d\tau) > \gamma - \beta/(1+r). \qquad \text{(C4)}$$

C.3 Steady state welfare effects

The steady state welfare results can be easily derived by letting t go to infinity in conditions (C2) - (C4).

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